

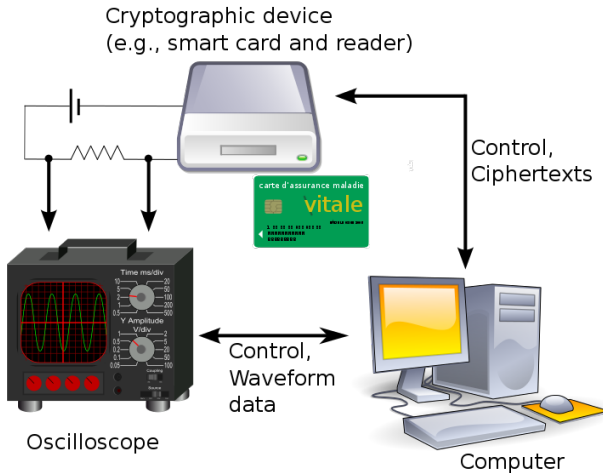
Higher Order Masking of Look-up Tables

Jean-Sébastien Coron

University of Luxembourg

EUROCRYPT, 2014-05-14

Side-channel Attacks



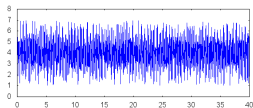
Differential Power Analysis [KJJ99]

Group by predicted
SBox output bit

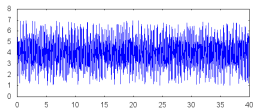
111



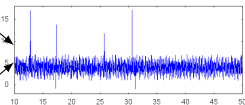
Average trace



000



Differential trace



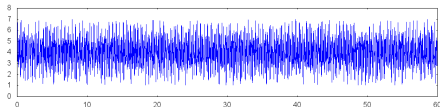
Masking Countermeasure

- Let x be some variable in a block-cipher.
- Masking countermeasure: generate a random r , and manipulate the masked value x'

$$x' = x \oplus r$$

instead of x .

- r is random $\Rightarrow x'$ is random
 \Rightarrow power consumption of x' is random



\Rightarrow no information about x is leaked

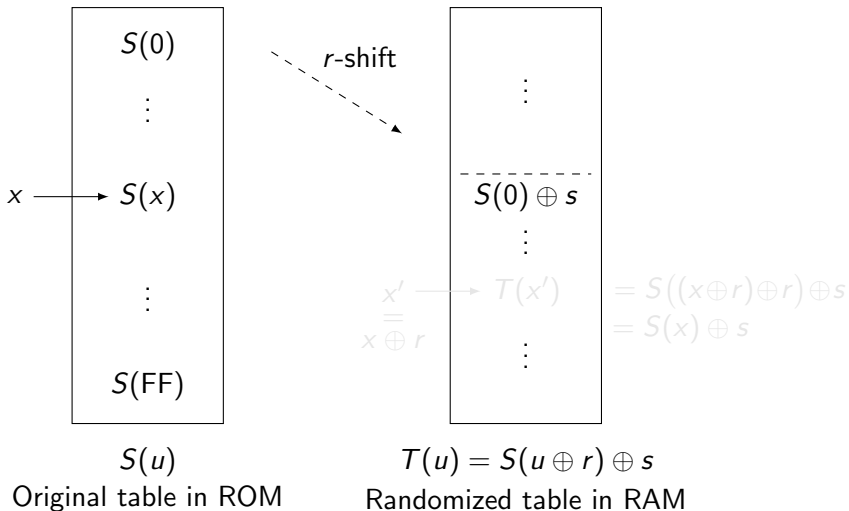
Masking Countermeasure

- How do we compute with $x' = x \oplus r$ instead of x ?
- Linear operation $f(x)$ (e.g. MixColumns in AES): easy

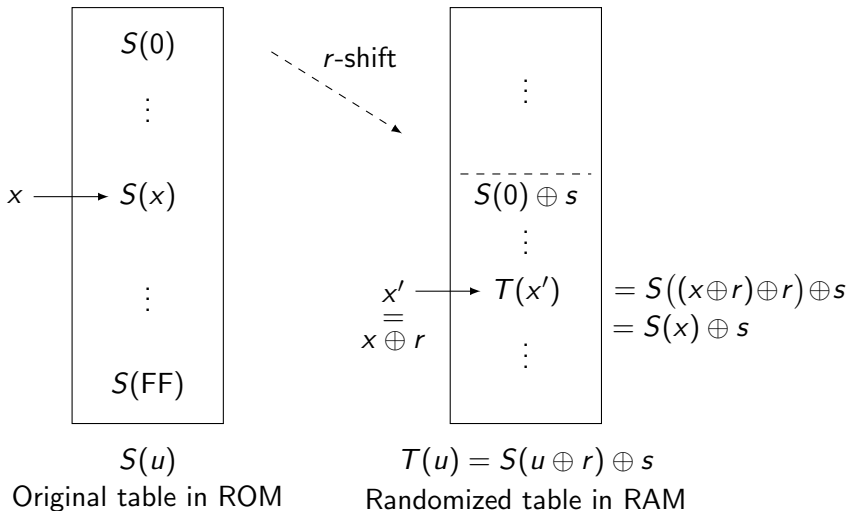
$$f(x') = f(x) \oplus f(r)$$

- We compute $f(x')$ and $f(r)$ separately.
- $f(x)$ is now masked with $f(r)$ instead of r .
- Non-linear operations (SBOX): randomized table [CJRR99]

Randomized Table Countermeasure [CJRR99]

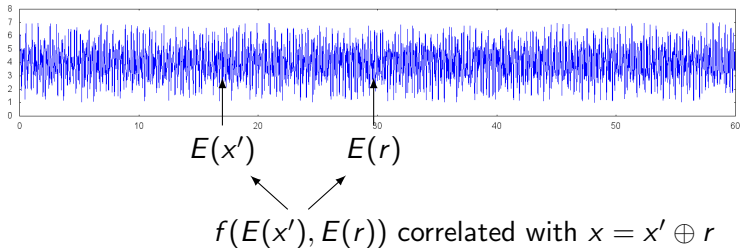


Randomized Table Countermeasure [CJRR99]



Second-order Attack

- Second-order attack:



- Requires more curves but can be practical

Higher-order masking

- Solution: n shares instead of 2:

$$x = x_1 \oplus x_2 \oplus \cdots \oplus x_n$$

- Any subset of $n - 1$ shares is uniformly and independently distributed
 - If we probe at most $n - 1$ shares x_i , we learn nothing about $x \Rightarrow$ secure against a DPA attack of order $n - 1$.
- Linear operations: still easy
 - Compute the $f(x_i)$ separately

$$f(x) = f(x_1) \oplus f(x_2) \oplus \cdots \oplus f(x_n)$$

Higher-order computation of SBoxes

- SBox computation ?
 - We have input shares x_1, \dots, x_n , with

$$x = x_1 \oplus x_2 \oplus \dots \oplus x_n$$

- We must output shares y_1, \dots, y_n , such that

$$S(x) = y_1 \oplus y_2 \oplus \dots \oplus y_n$$

- without leaking information about x .
- **This talk:** first generalization of the previous randomized table countermeasure to n shares.

Existing Higher Order Countermeasure

- Ishai-Sahai-Wagner private circuit [ISW03]
 - Shows how to transform any boolean circuit C into a circuit of size $\mathcal{O}(|C| \cdot t^2)$ perfectly secure against t probes.
- Rivain-Prouff (CHES 2010) countermeasure for AES:

$$S(x) = x^{254} \in \mathbb{F}_{2^8}$$

- Secure multiplication based on [ISW03]:

$$z = xy = \left(\bigoplus_{i=1}^n x_i \right) \cdot \left(\bigoplus_{i=1}^n y_i \right) = \bigoplus_{1 \leq i, j \leq n} x_i y_j$$

- Provably secure against t -th order DPA with $n \geq 2t + 1$ shares.

Existing Higher Order Countermeasures

- Carlet *et al.* (FSE 2012) countermeasure for any Sbox.
 - Lagrange interpolation

$$S(x) = \sum_{i=0}^{2^k-1} \alpha_i \cdot x^i$$

over \mathbb{F}_{2^k} , for constant coefficients $\alpha_i \in \mathbb{F}_{2^k}$.

- **This talk:** alternative to Rivain-Prouff and Carlet *et al.* countermeasures
 - Generalization of the classical randomized table countermeasure.
 - No field operations, only table recomputation.

Existing Higher Order Countermeasures

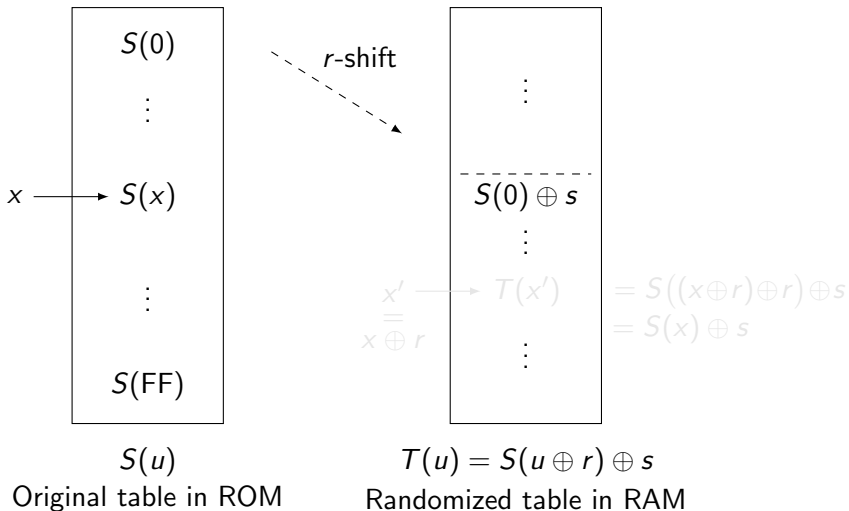
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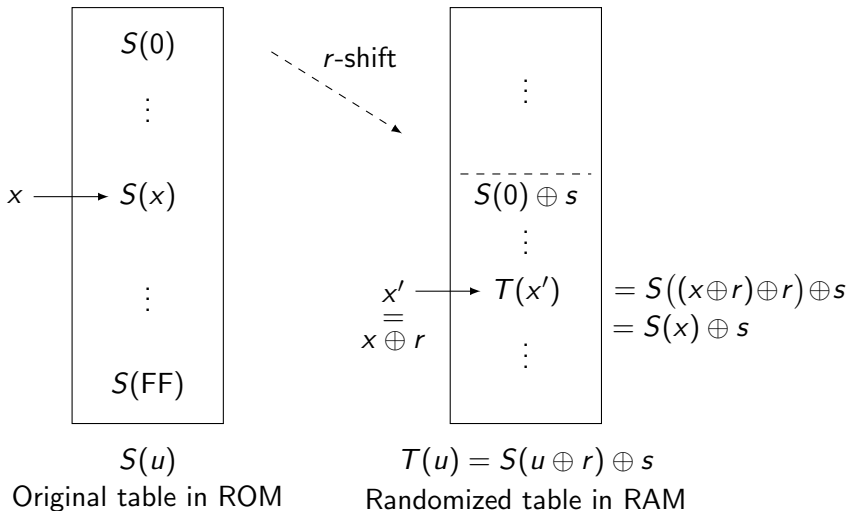
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Randomized Table Countermeasure [CJRR99]

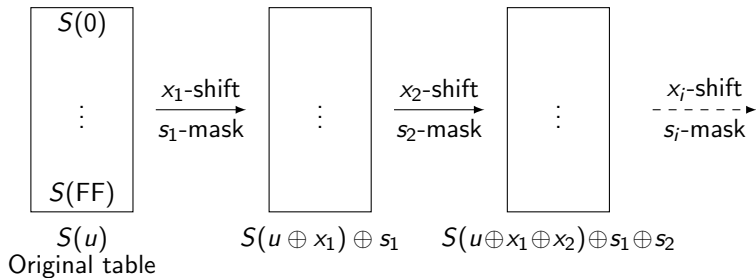


Randomized Table Countermeasure [CJRR99]



First attempt: Schramm and Paar countermeasure [SP06]

$$x = x_1 \oplus x_2 \oplus \dots \oplus x_n$$

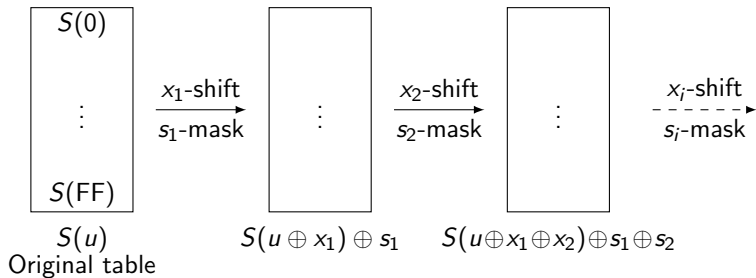


$$x_n = x \oplus x_1 \oplus \dots \oplus x_{n-1} \rightarrow T(x_n) = S(x) \oplus s_1 \oplus \dots \oplus s_{n-1}$$

$$T(u) = S(u \oplus x_1 \oplus \dots \oplus x_{n-1}) \oplus s_1 \oplus \dots \oplus s_{n-1}$$

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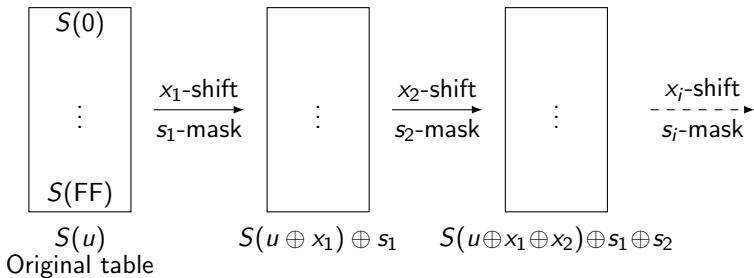


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Third-order Attack for any n

- Final randomized table:

$T(0)$	$= S(0 \oplus x_1 \oplus \dots \oplus x_{n-1}) \oplus s_1 \oplus \dots \oplus s_{n-1}$
$T(1)$	$= S(1 \oplus x_1 \oplus \dots \oplus x_{n-1}) \oplus s_1 \oplus \dots \oplus s_{n-1}$
\vdots	$= S(0 \oplus x_1 \oplus \dots \oplus x_{n-1}) \oplus S(1 \oplus x_1 \oplus \dots \oplus x_{n-1})$

only depends on $x_1 \oplus \dots \oplus x_{n-1}$,
also probe $x_n \Rightarrow$ 3rd order attack.

- For high-order countermeasures, do not reuse the same masks multiple times !
 - Using the same mask r is OK only for first-order countermeasures.

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New Countermeasure

- **This talk:** new countermeasure for SBOXes, secure against higher-order attacks:
 - Variant of Schramm and Paar countermeasure
 - but use different masks for every line of the Sbox
 - and refresh the masks between successive shifts of the table.
- Provably secure against t -th order DPA, in the ISW model, with $n \geq 2t + 1$ shares.
 - Alternative to Rivain-Prouff and Carlet *et al.* countermeasures based on finite-fields operations.

Initial table with n shares

- Every line of the SBox is initially randomly shared among n shares, independently for every line.

$(s_{00,1}, \dots, s_{00,n})$	$S(00)$
\vdots	
$(s_{u,1}, \dots, s_{u,n})$	$S(u)$
\vdots	
$(s_{FF,1}, \dots, s_{FF,n})$	$S(FF)$

Original shared table

- Equivalent to having n randomized tables instead of 1.
- The lines of the table are then progressively shifted by x_1, x_2, \dots, x_{n-1} , as in Schramm and Paar, but with a RefreshMask after every shift.

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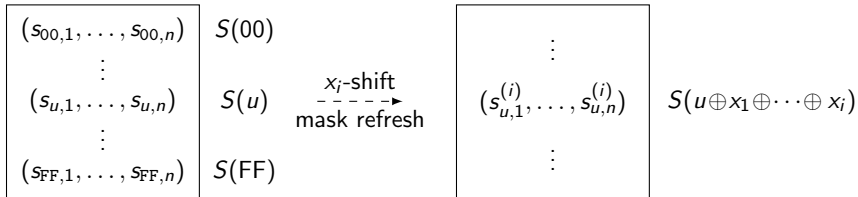
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Original shared table

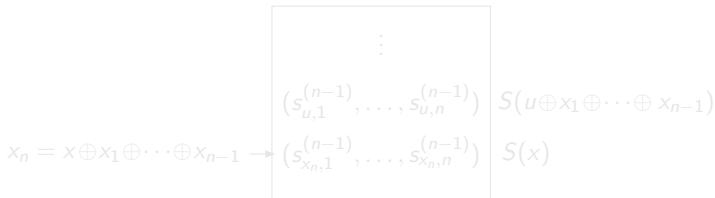
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Iterative input shift by x_i

$$x = x_1 \oplus x_2 \oplus \dots \oplus x_n$$



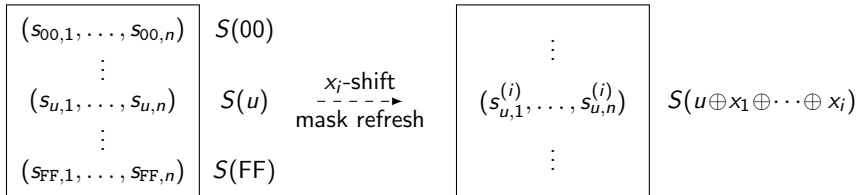
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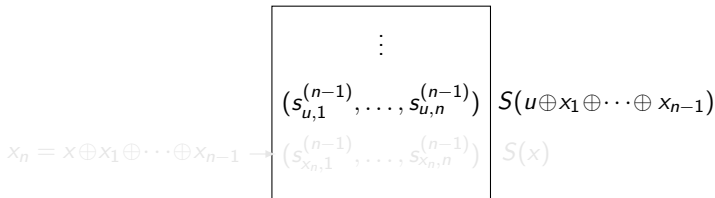
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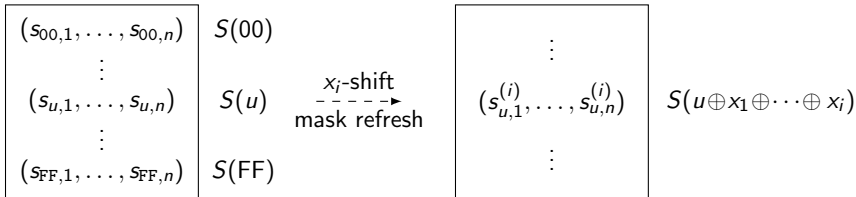
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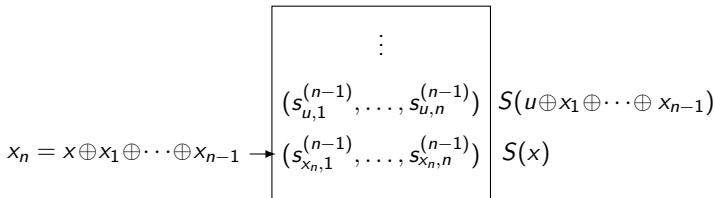
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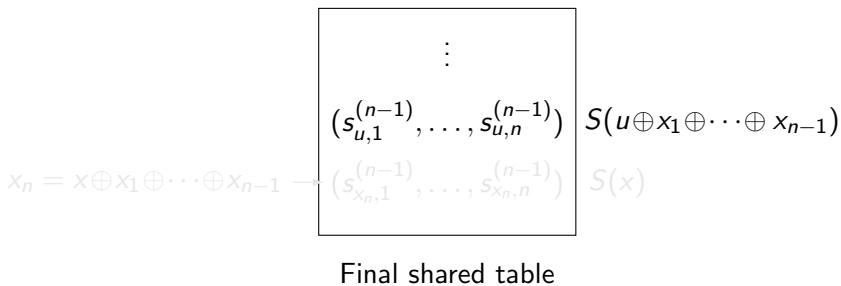
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Final shared table

Final randomized table

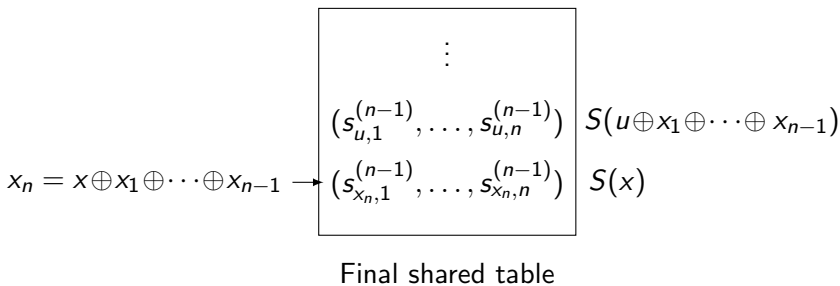
- In the final shared table, the inputs are shifted by $x_1 \oplus \dots \oplus x_{n-1}$:



- The n output shares $T(x_n) = (s_{x_n,1}^{(n-1)}, \dots, s_{x_n,n}^{(n-1)})$ correspond to the output $S(x)$

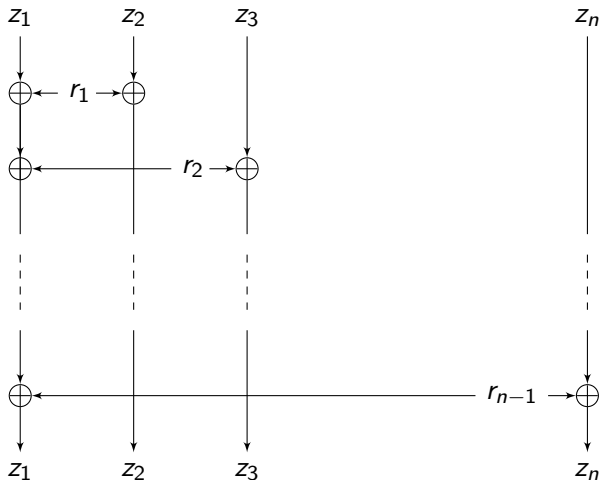
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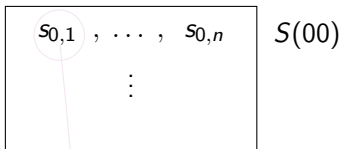
Mask Refreshing



- Required property: any subset of $n - 1$ output shares z_i is uniformly and independently distributed.

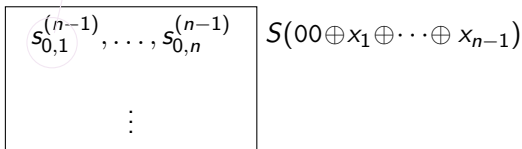
Why are the mask refreshing necessary ?

- Without mask refreshing:



Original shared table

If collision, then presumably $x_1 \oplus \dots \oplus x_{n-1} = 0$
also probe $x_n \Rightarrow$ 3rd order attack

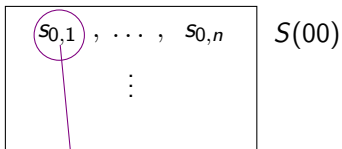


Final shared table

- The mask refreshing prevents from correlating shares between different shifts of the tables.

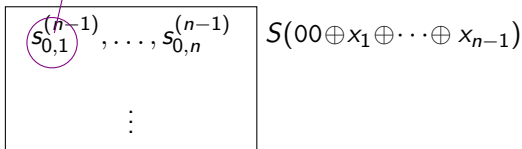
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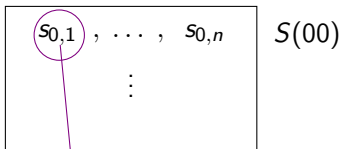


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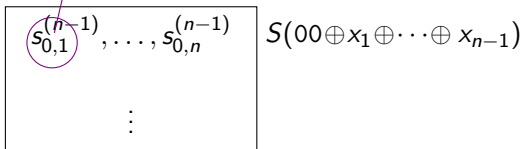
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Full Algorithm

Algorithm 1 Masked computation of $y = S(x)$

Input: x_1, \dots, x_n such that $x = x_1 \oplus \dots \oplus x_n$

Output: y_1, \dots, y_n such that $y = S(x) = y_1 \oplus \dots \oplus y_n$

```
1: for all  $u \in \{0, 1\}^k$  do
2:    $T(u) \leftarrow (S(u), 0, \dots, 0) \in (\{0, 1\}^{k'})^n$   $\triangleright \oplus(T(u)) = S(u)$ 
3: end for
4: for  $i = 1$  to  $n - 1$  do
5:   for all  $u \in \{0, 1\}^k$  do
6:     for  $j = 1$  to  $n$  do  $T'(u)[j] \leftarrow T(u \oplus x_i)[j]$   $\triangleright T'(u) \leftarrow T(u \oplus x_i)$ 
7:   end for
8:   for all  $u \in \{0, 1\}^k$  do
9:      $T(u) \leftarrow \text{RefreshMasks}(T'(u))$   $\triangleright \oplus(T(u)) = S(u \oplus x_1 \oplus \dots \oplus x_i)$ 
10:  end for
11: end for  $\triangleright \oplus(T(u)) = S(u \oplus x_1 \oplus \dots \oplus x_{n-1})$  for all  $u \in \{0, 1\}^k$ .
12:  $(y_1, \dots, y_n) \leftarrow \text{RefreshMasks}(T(x_n))$   $\triangleright \oplus(T(x_n)) = S(x)$ 
13: return  $y_1, \dots, y_n$ 
```

Mask refreshing

Algorithm 2 RefreshMasks

Input: z_1, \dots, z_n such that $z = z_1 \oplus \dots \oplus z_n$

Output: z_1, \dots, z_n such that $z = z_1 \oplus \dots \oplus z_n$

- 1: **for** $i = 2$ to n **do**
 - 2: $tmp \leftarrow \{0, 1\}^{k'}$
 - 3: $z_1 \leftarrow z_1 \oplus tmp$
 - 4: $z_i \leftarrow z_i \oplus tmp$
 - 5: **end for**
 - 6: **return** z_1, \dots, z_n
-

Asymptotic Complexity

- Asymptotic complexity for k -bit SBox and n shares:

Countermeasure	Time comp.	Memory comp.
Carlet <i>et al.</i>	$\mathcal{O}(2^{k/2} \cdot n^2)$	$\mathcal{O}(2^{k/2} \cdot n)$
New countermeasure.	$\mathcal{O}(2^k \cdot n^2)$	$\mathcal{O}(2^k \cdot n)$
New count. (large register)	$\mathcal{O}(2^{k/2} \cdot n^2)$	$\mathcal{O}(2^k \cdot n)$

- Large register variant: pack multiple Sbox outputs in a single register
 - For DES, pack 8 output 4-bit nibbles into a 32-bit register
 - Running time divided by 8

Asymptotic Complexity

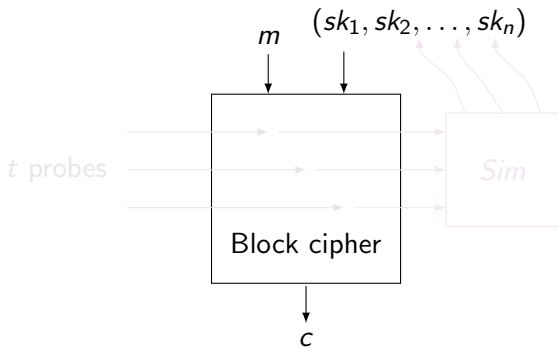
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ISW security model

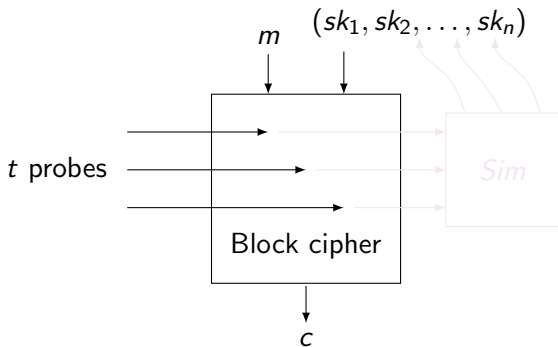
- Simulation framework of [ISW03]:



- Show that any t probes can be perfectly simulated from at most $n - 1$ of the sk_i 's.
- Those $n - 1$ shares sk_i are initially uniformly and independently distributed.
- \Rightarrow the adversary learns nothing from the t probes, since he could perfectly simulate those t probes by himself.

ISW security model

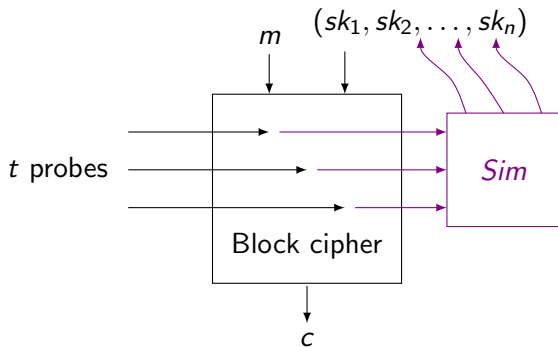
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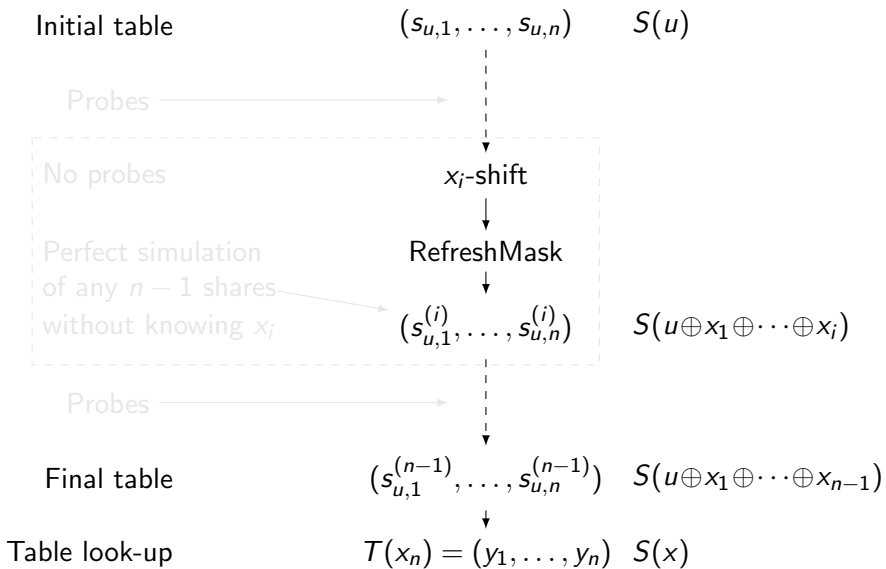
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Security of high-order table recomputation

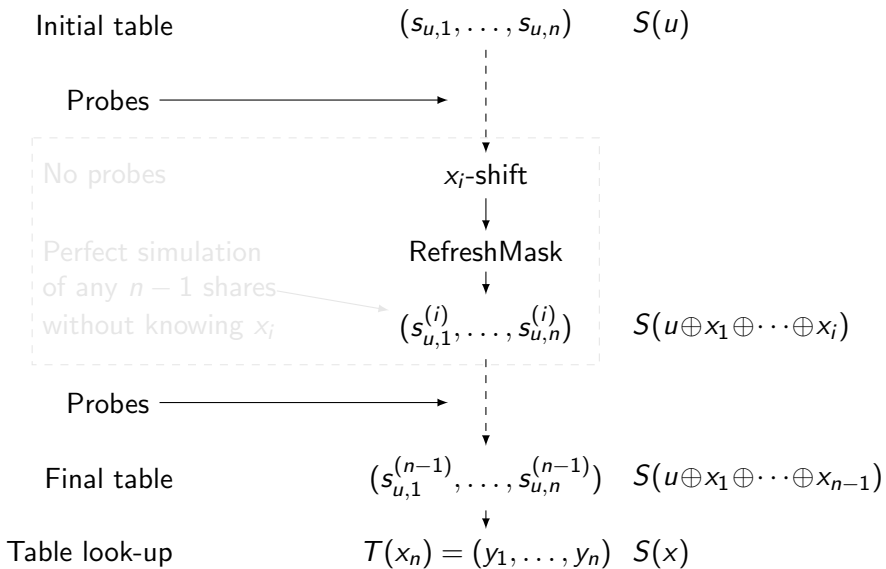
Theorem

The table recomputation countermeasure is secure against t probes in the ISW model, for $n \geq 2t + 1$.

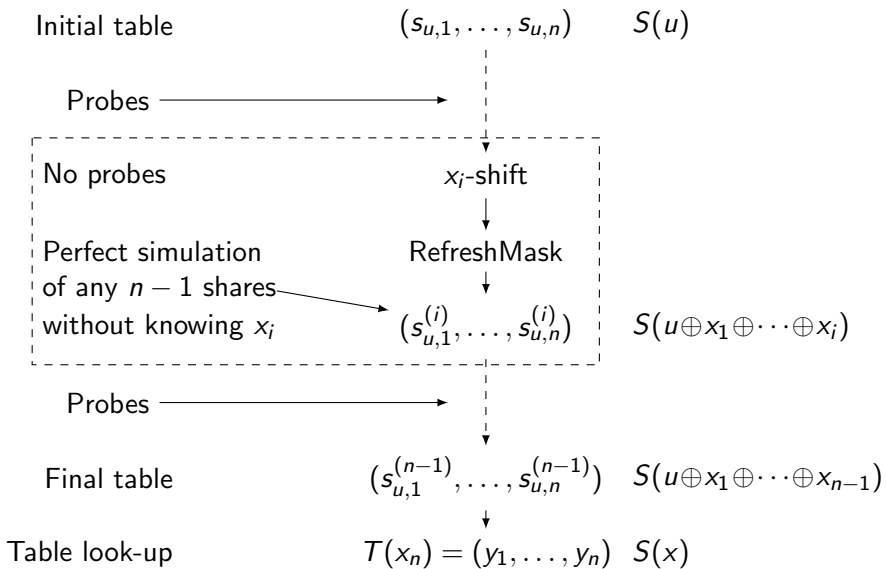
Proof sketch



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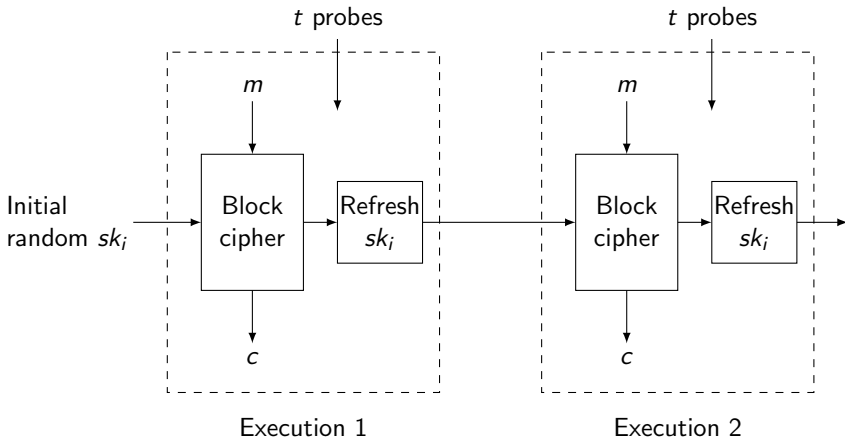


Proof sketch



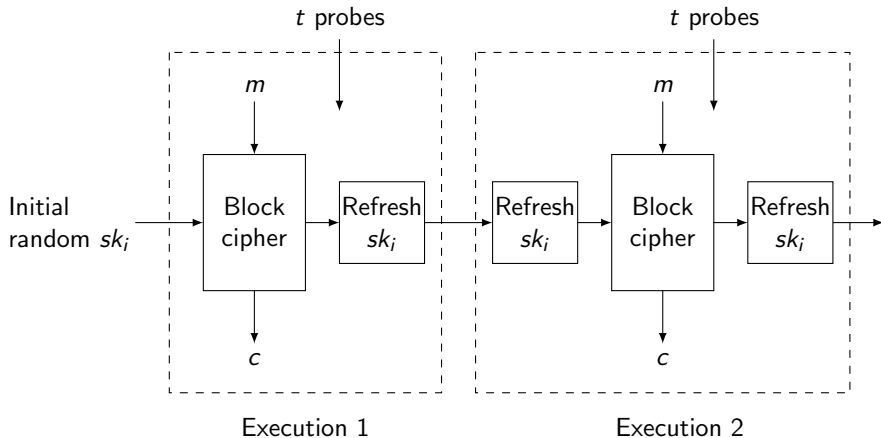
Protecting a full block-cipher

- Adaptive model of [ISW03]:
 - The adversary can move its t probes between successive executions of the block-cipher.
 - $n \geq 4t + 1$ are sufficient to guarantee security in the adaptive model



Protecting a full block-cipher

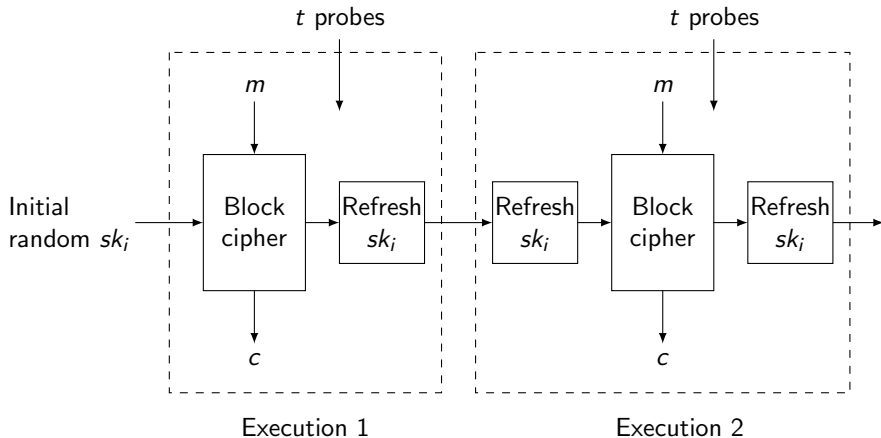
- Improvement: $n \geq 2t + 1$ are sufficient to guarantee security in the adaptive model



- Optimal: \mathcal{A} can probe t shares sk_i at the end of one execution and t shares sk_i at the beginning of the next.

Protecting a full block-cipher

- Improvement: $n \geq 2t + 1$ are sufficient to guarantee security in the adaptive model



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Performances for AES

	t	n	Time (ms)	Penalty
AES, unmasked			0.0018	1
AES, Rivain-Prouff	1	3	0.092	50
AES, table recomputation	1	3	0.80	439
AES, Rivain-Prouff	2	5	0.18	96
AES, table recomputation	2	5	2.2	1205
AES, Rivain-Prouff	3	7	0.31	171
AES, table recomputation	3	7	4.4	2411
AES, Rivain-Prouff	4	9	0.51	276
AES, table recomputation	4	9	7.3	4003

- Table recomputation an order of magnitude slower than RP
 - RP can take advantage of the special structure of the AES SBox (only 4 mults in \mathbb{F}_{2^8}).

Performances for DES

	t	n	Time (ms)	Penalty
DES, unmasked			0.010	1
DES, Carlet <i>et al.</i>	1	3	0.47	47
DES, table recomputation	1	3	0.31	31
DES, Carlet <i>et al.</i>	2	5	0.78	79
DES, table recomputation	2	5	0.59	59
DES, Carlet <i>et al.</i>	3	7	1.3	129
DES, table recomputation	3	7	0.90	91
DES, Carlet <i>et al.</i>	4	9	1.9	189
DES, table recomputation	4	9	1.4	142

- For DES the performances are similar
- <http://github.com/coron/htable/>

Questions ?