Déjà Q: Using Dual Systems to Revisit q-Type Assumptions

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Sarah Meiklejohn (UC San Diego → University College London)

Pairing-based cryptography: a brief history

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Even later assumptions: q-SDH, q-ADHSDH, q-EDBDH, q-SDH-III, q-SFP, “source group q-parallel BDHE,” etc.

Why are q-type assumptions worrisome?

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t/√q steps [Cheon06]

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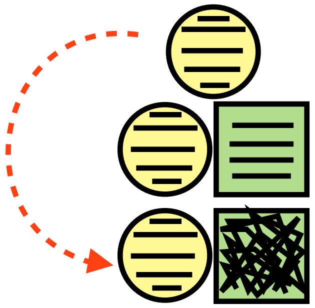
- Abstract dual systems into three steps



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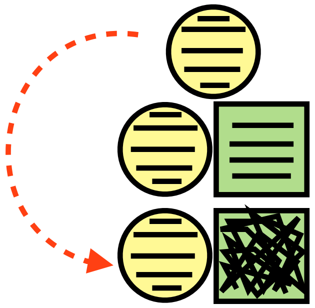

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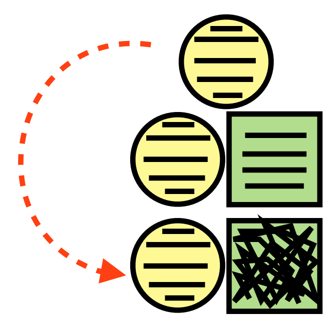

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- Reduce* to an assumption that holds by a statistical argument

*currently only in composite-order groups

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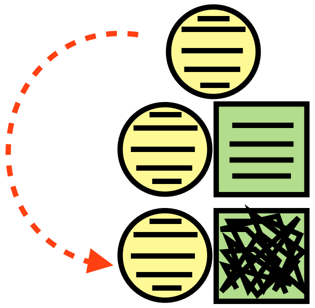

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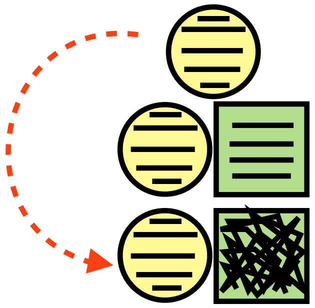

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Extension to Dodis-Yampolskiy PRF [DY05]

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Outline

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Standard bilinear group: (N, G, H, G_T, e, g, h)

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$e: G \times H \to G_T$

bilinearity: $e(g^a,h^b) = e(g,h)^{ab}\ \forall a,b \in Z/NZ$

non-degeneracy: $e(x,y) = 1\ \forall y \in H \Rightarrow x = 1$

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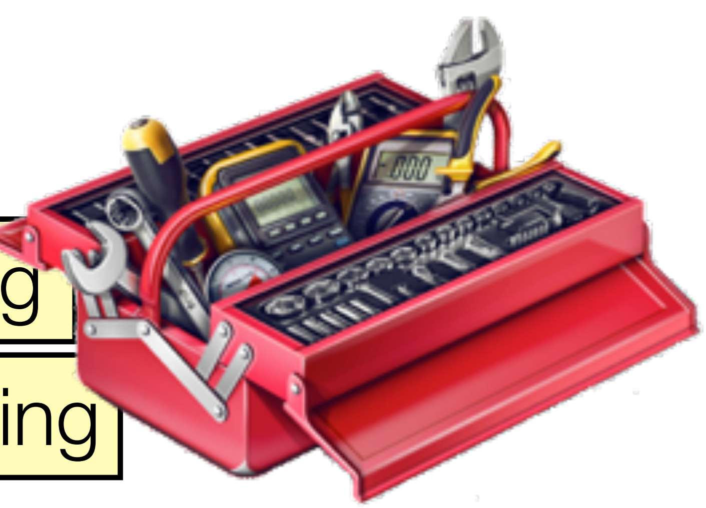
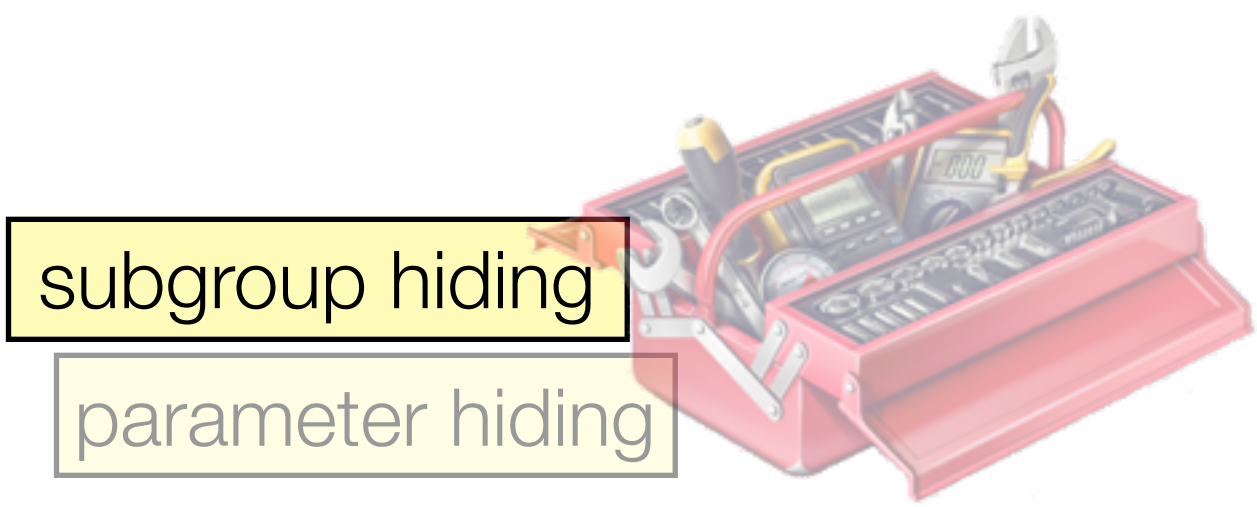


Subgroup hiding

Composite-order bilinear group: (N, G, G_T, e, g) where $N = pq$



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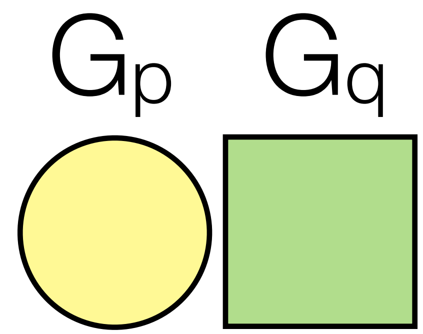

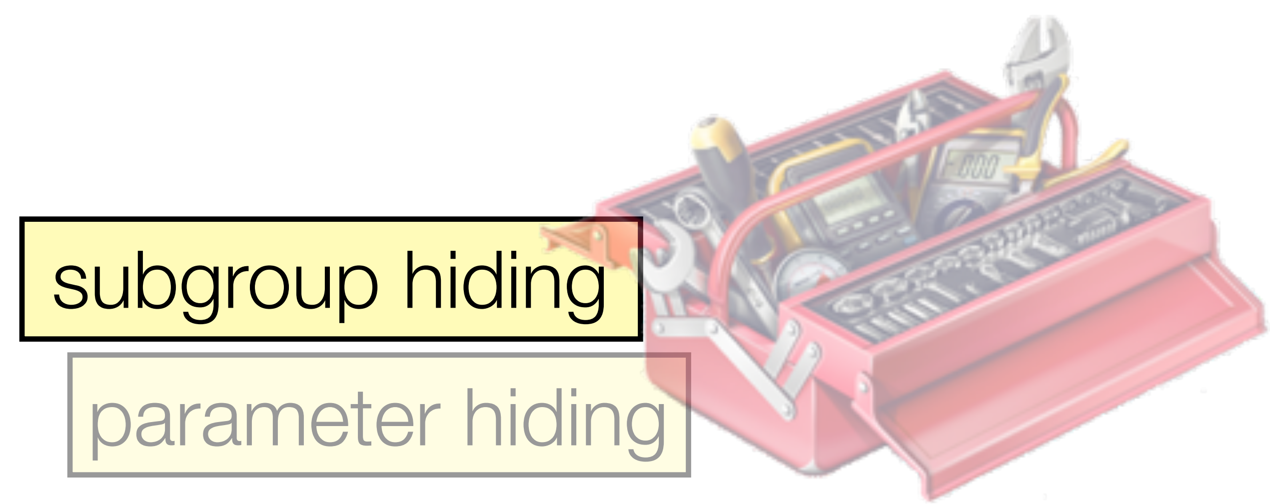

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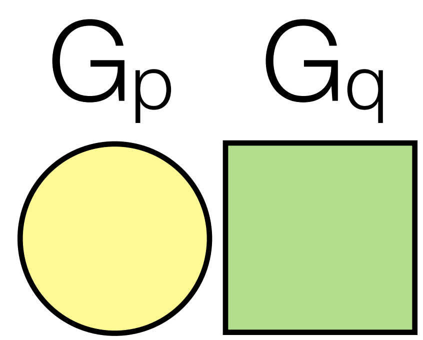

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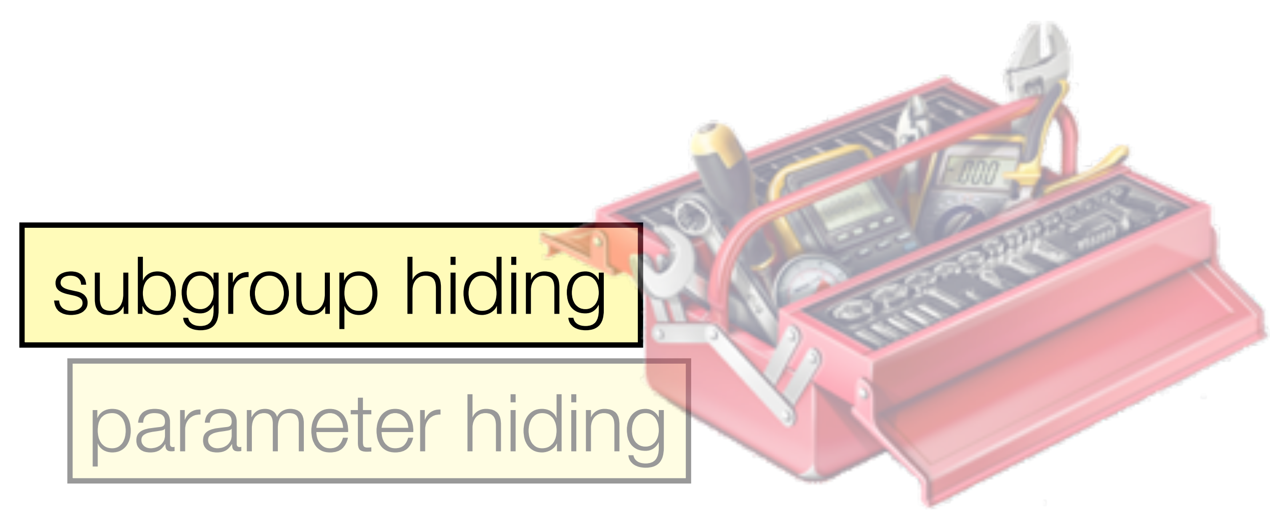

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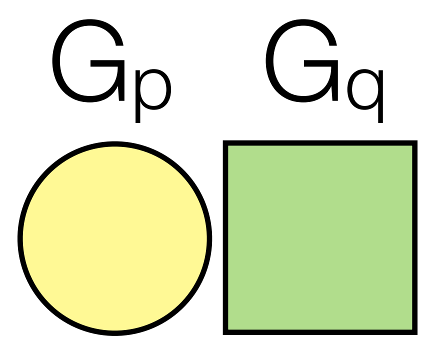

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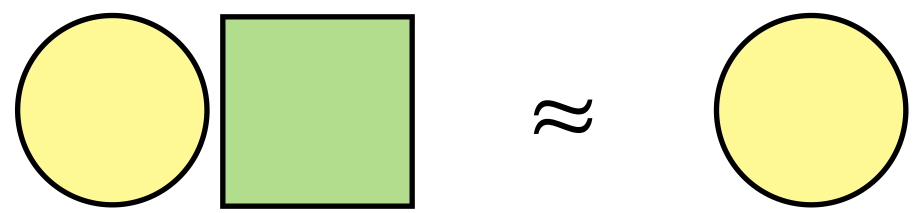

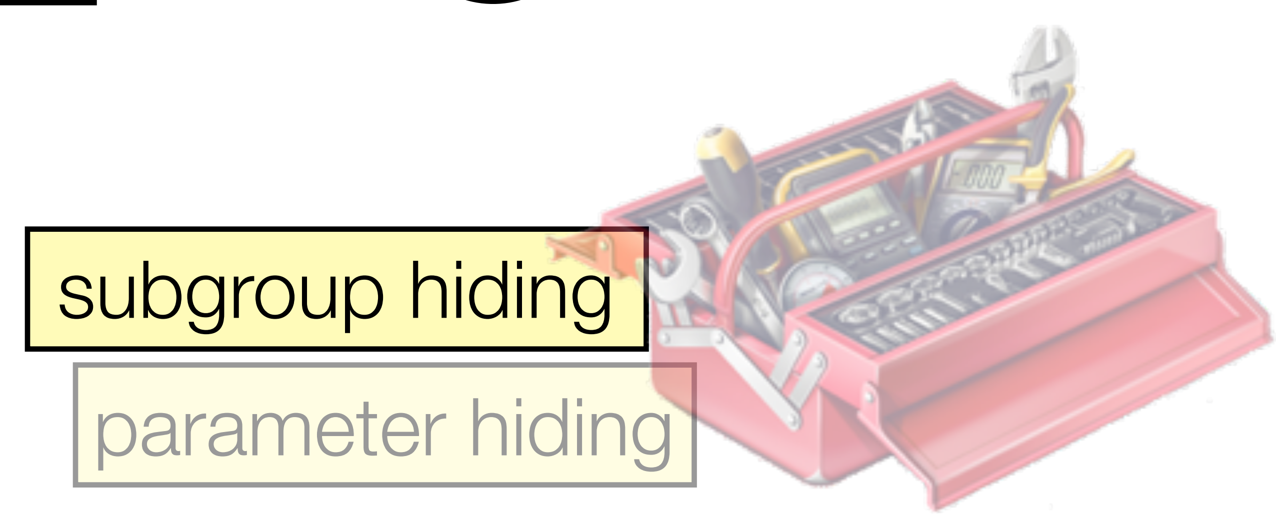

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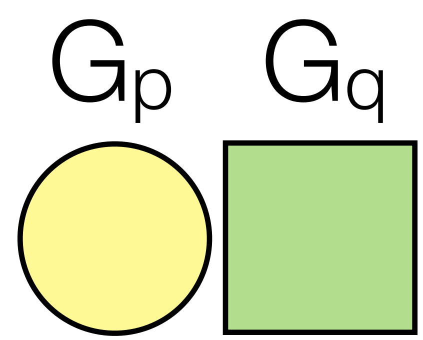

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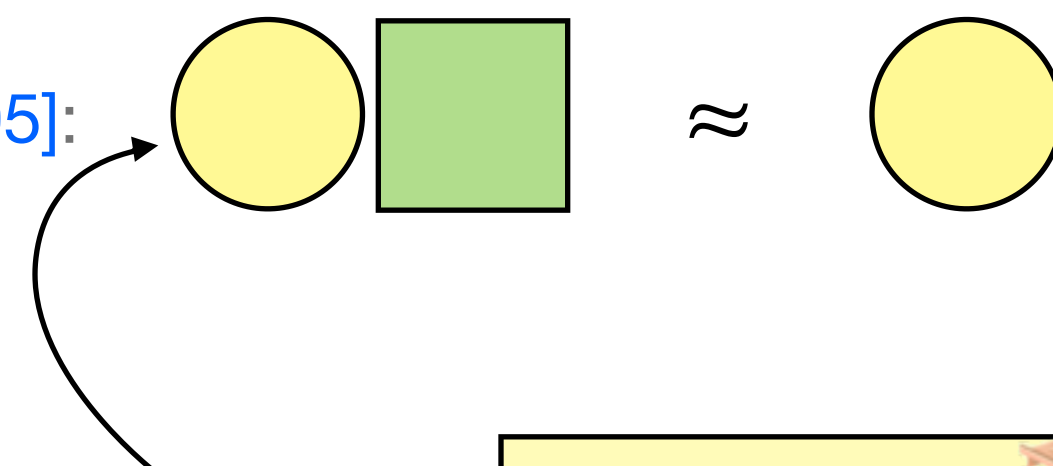
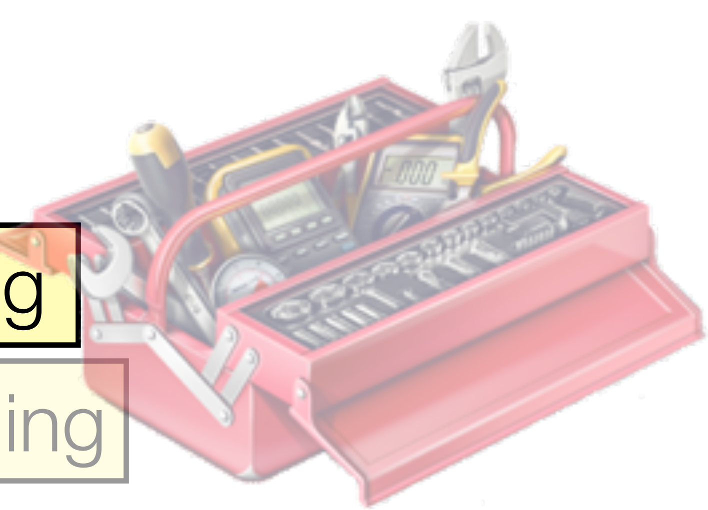


random element of $G_p \times G_q$

subgroup hiding

parameter hiding



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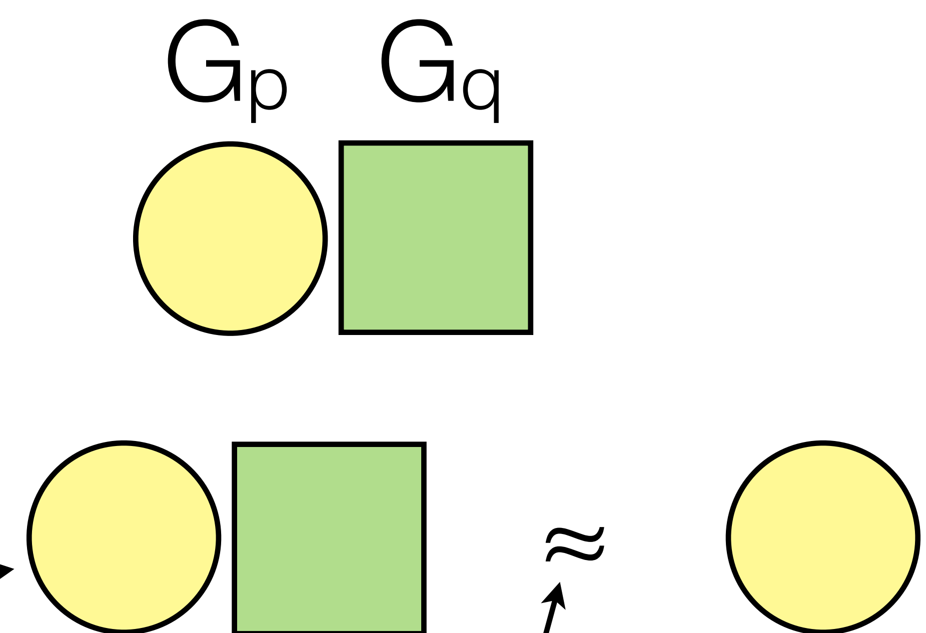

(indistinguishable from)

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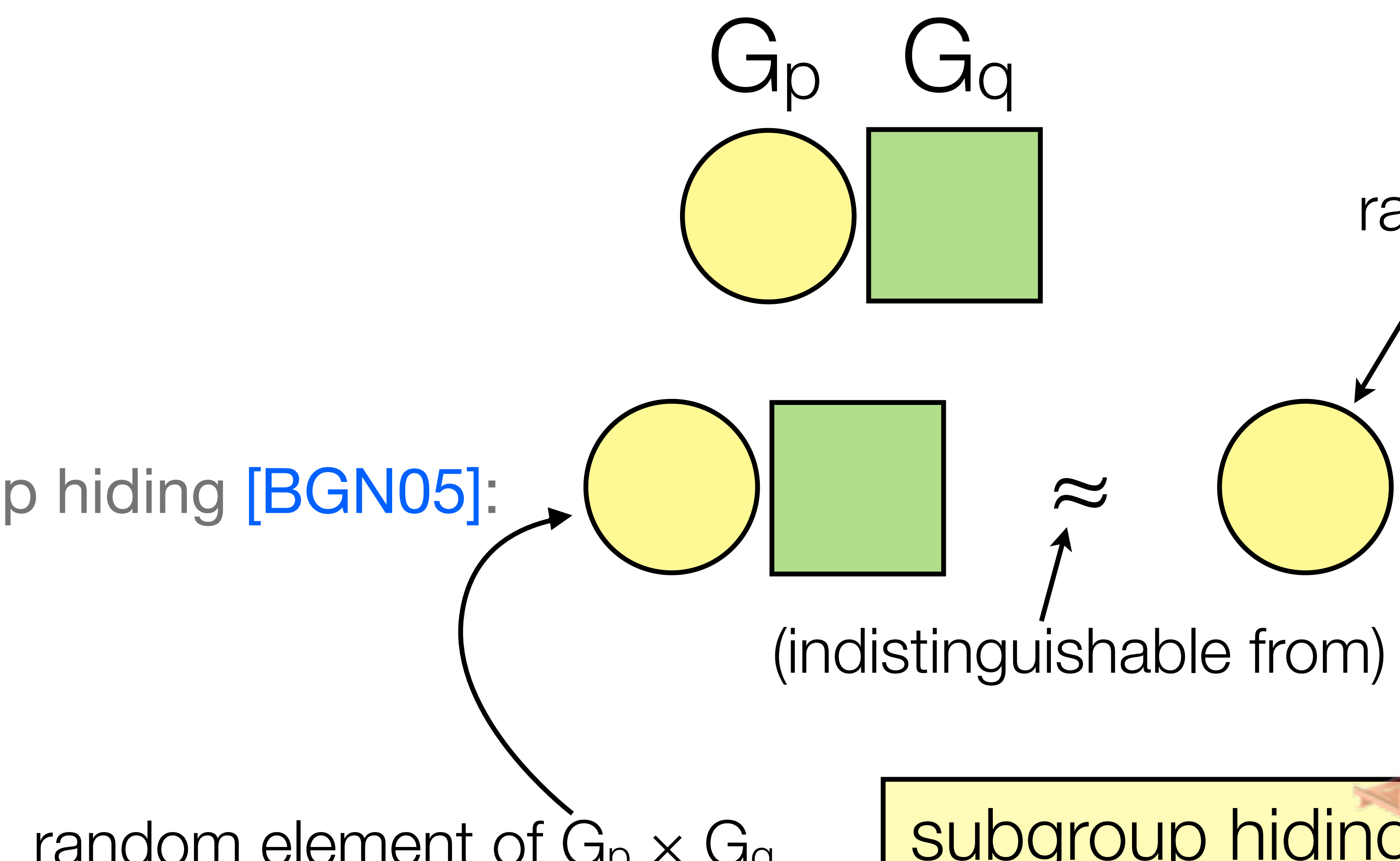

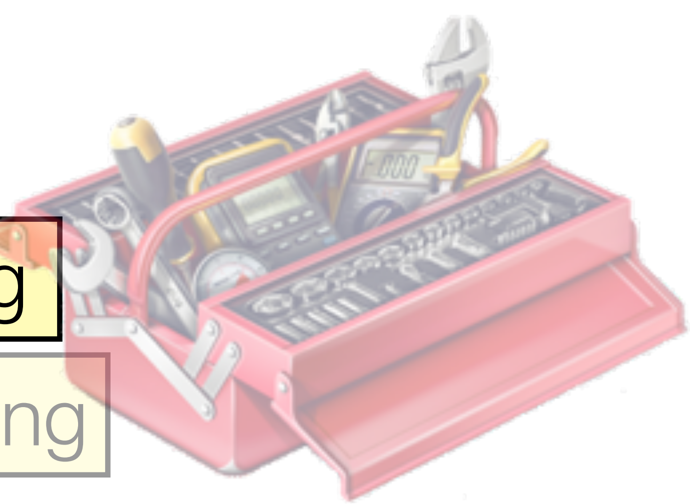

Parameter hiding [L12]

Parameter hiding: elements correlated across subgroups are distributed identically to uncorrelated elements

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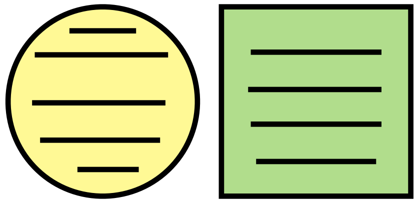

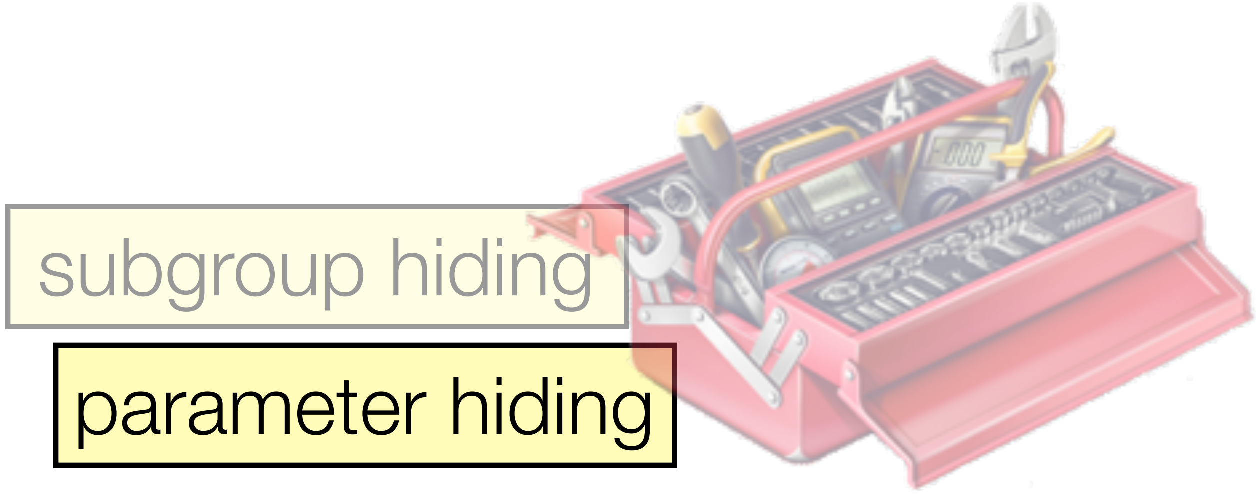

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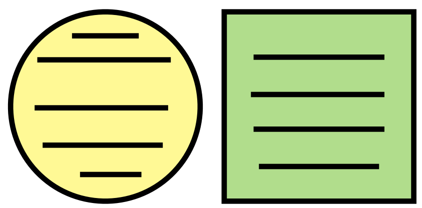

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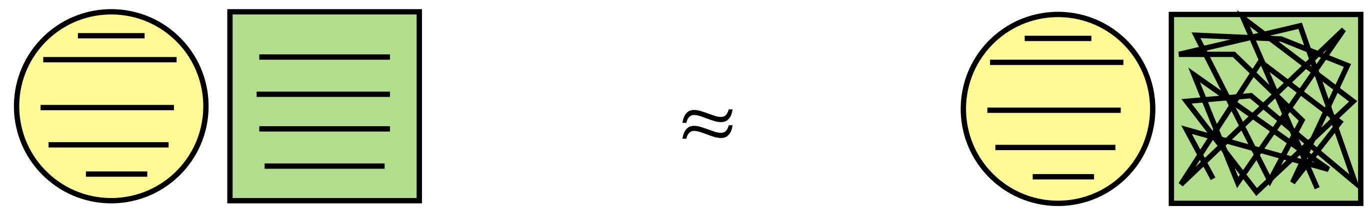

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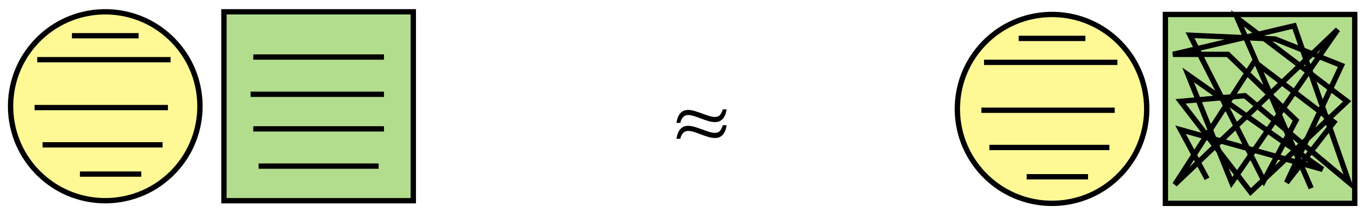

$$g_1^{f(x_1,\ldots,x_c)} g_2^{f(x_1,\ldots,x_c)} \approx g_1^{f(x_1,\ldots,x_c)} g_2^{f(x_1',\ldots,x_c')}$$

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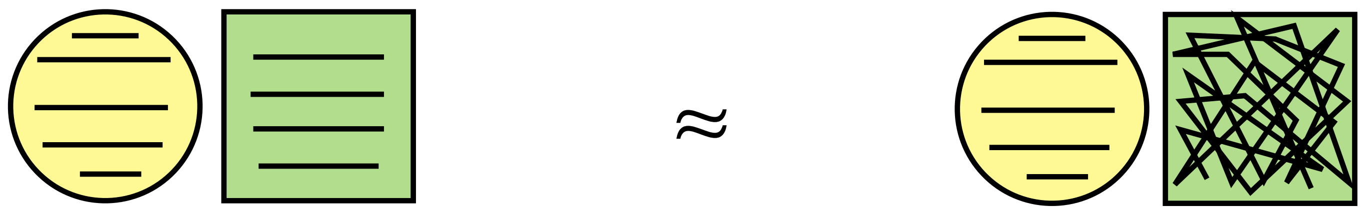

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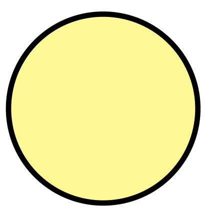
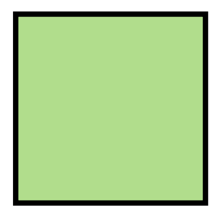
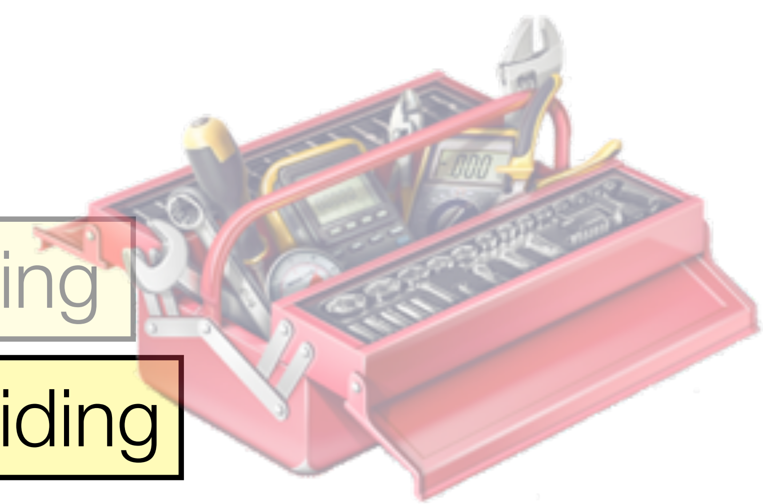
 is independent from 



subgroup hiding

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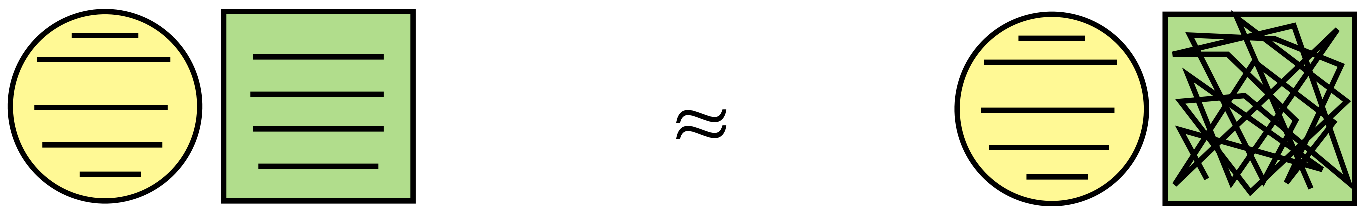

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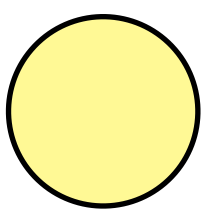
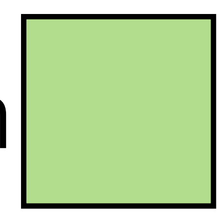
 is independent from 

x_i mod p reveals nothing about x_i mod q (CRT)

Typical dual-system proof for IBE [W09,LW10,...]

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Challenge ciphertext

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ID queries

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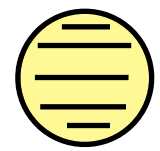
normal: 

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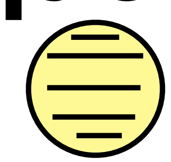
normal: 

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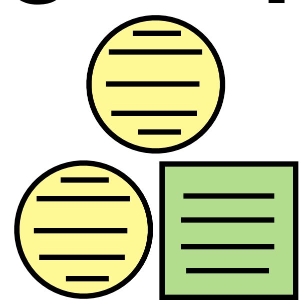

(subgroup hiding)

ID queries

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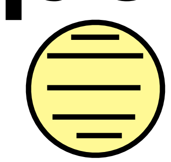

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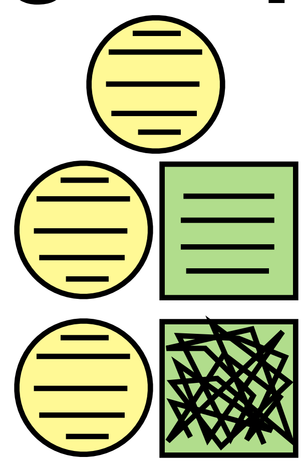

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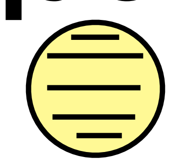

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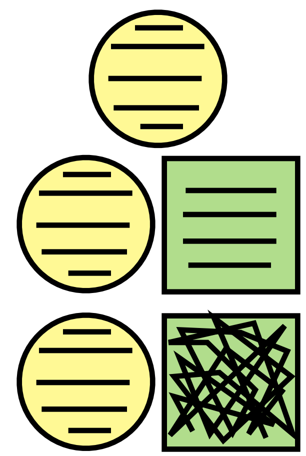

(subgroup hiding)

semi-functional (SF):

(parameter hiding)

ID queries

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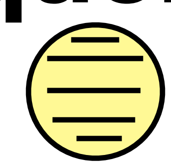

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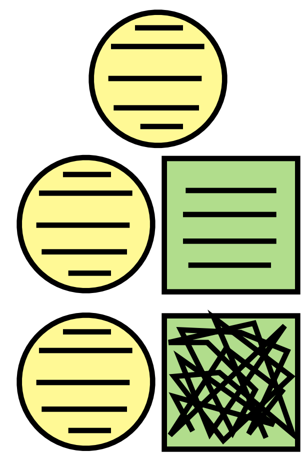

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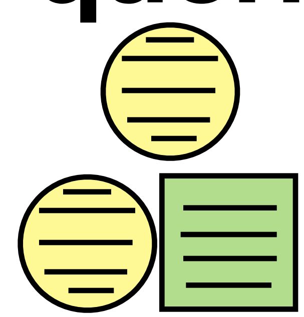

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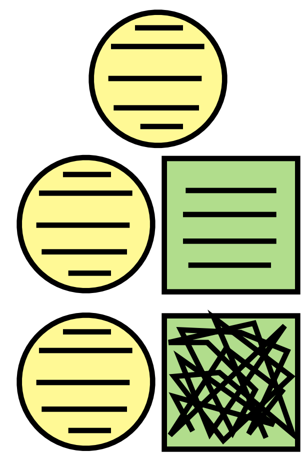

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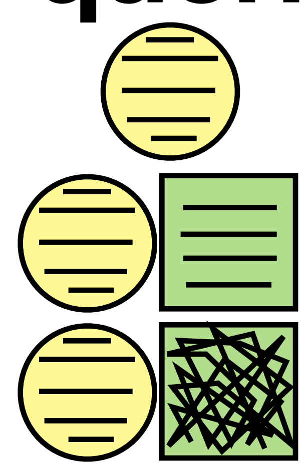

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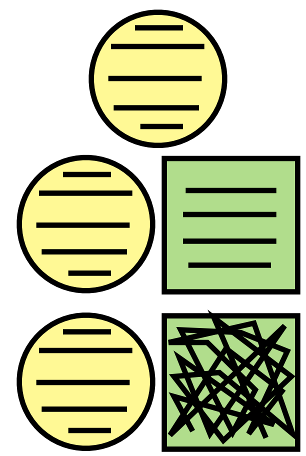

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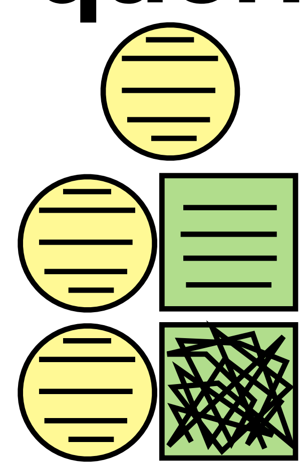

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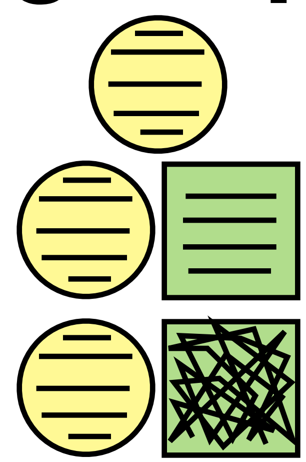

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SF keys don’t decrypt SF ciphertexts!

ID queries

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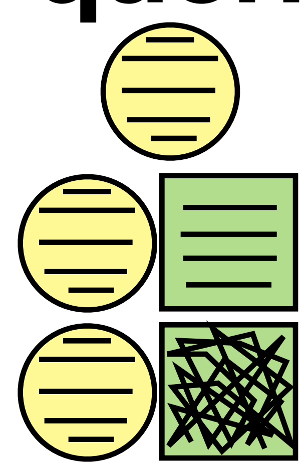

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Dual systems in three easy steps

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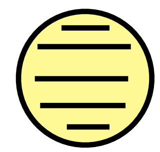
normal: 

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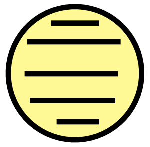

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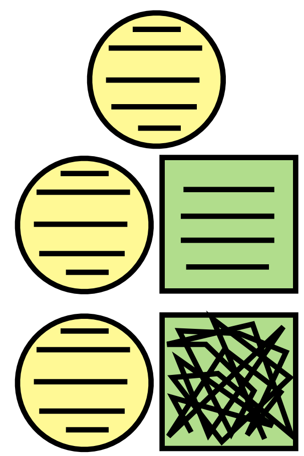

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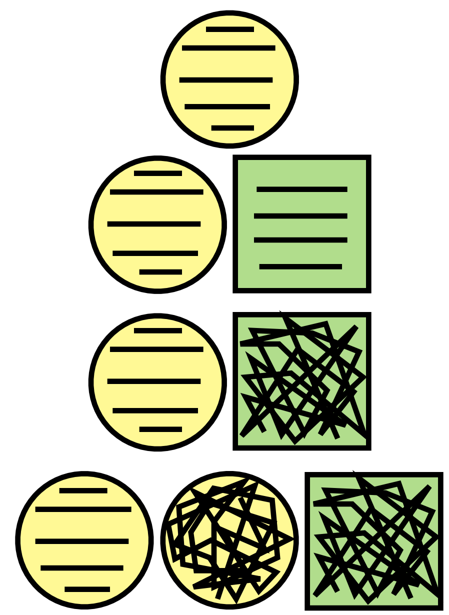

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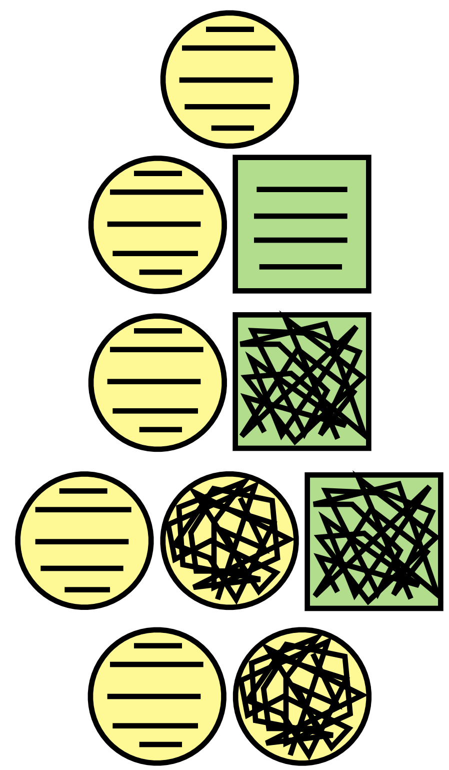

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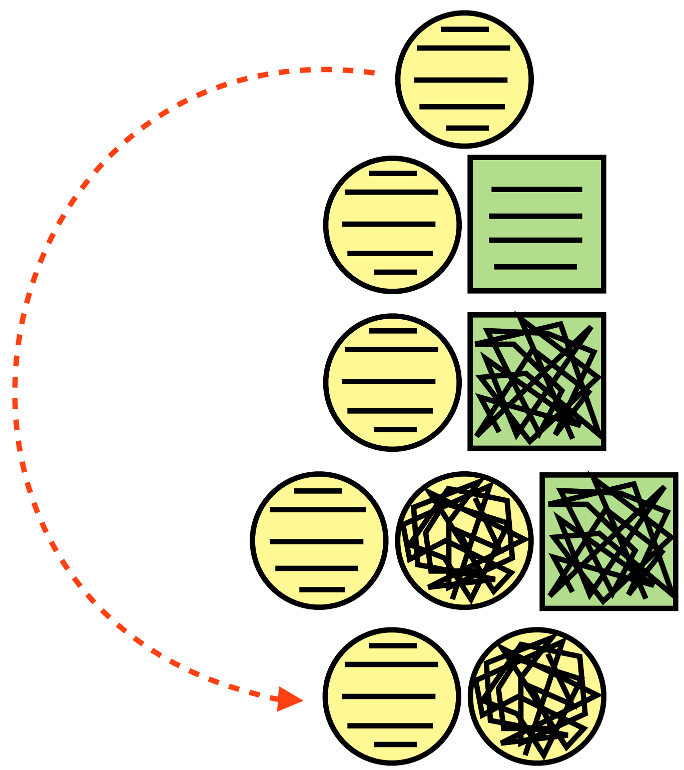

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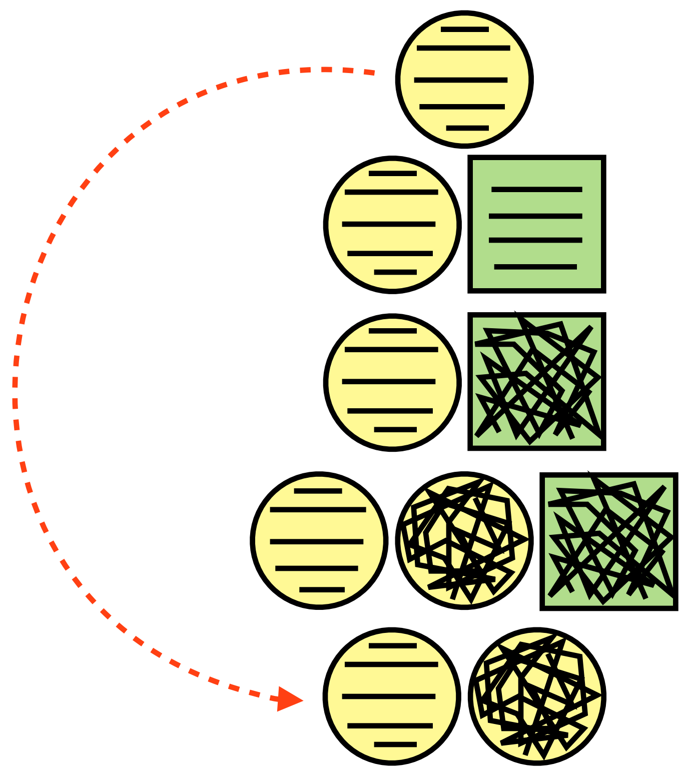

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2. transition to SF version
3. argue information is hidden

Outline

Bilinear groups

q-Type assumptions
The uber-assumption
Relating uber-assumptions
A bijection trick

Extensions

Conclusions

The “uber-assumption” [BBG05,B08]

Uber-assumption is parameterized by (c,R,S,T,f)

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uber(c,R,S,T,f) assumption: given (R,S,T) values, hard to compute/distinguish f

Example uber-assumption: exponent q-SDH

exponent q-SDH [ZS-NS04]: given $(g, g^{x}, \ldots, g^{x^q})$, distinguish $g^{x^{q+1}}$ from random

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exponent q-SDH is $uber(1,<1,\{x^i\}>,<1>,<1>,x^{q+1})$

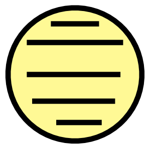
Applying dual systems to exponent q-SDH

uber(c,<1,$\{x^i\}$>,<1>,<1>,x^{q+1})

1. start with base scheme
2. transition to SF version
3. argue information is hidden

Applying dual systems to exponent q-SDH

$uber(c,<1,\{x^i\}>,<1>,<1>,x^{q+1})$



$g_1^{r_1 x_1},\ldots,g_1^{r_1 x_1^q}$

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Applying dual systems to exponent q-SDH

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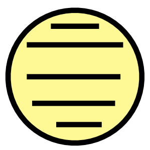

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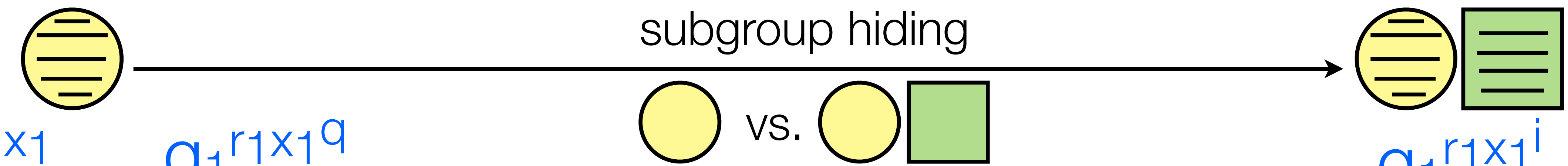

$g_1^{r_1x_1},\ldots,g_1^{r_1x_1^q}$

$g_1^{r_1x_1^i} \cdot g_2^{r_1'x_1^i}$

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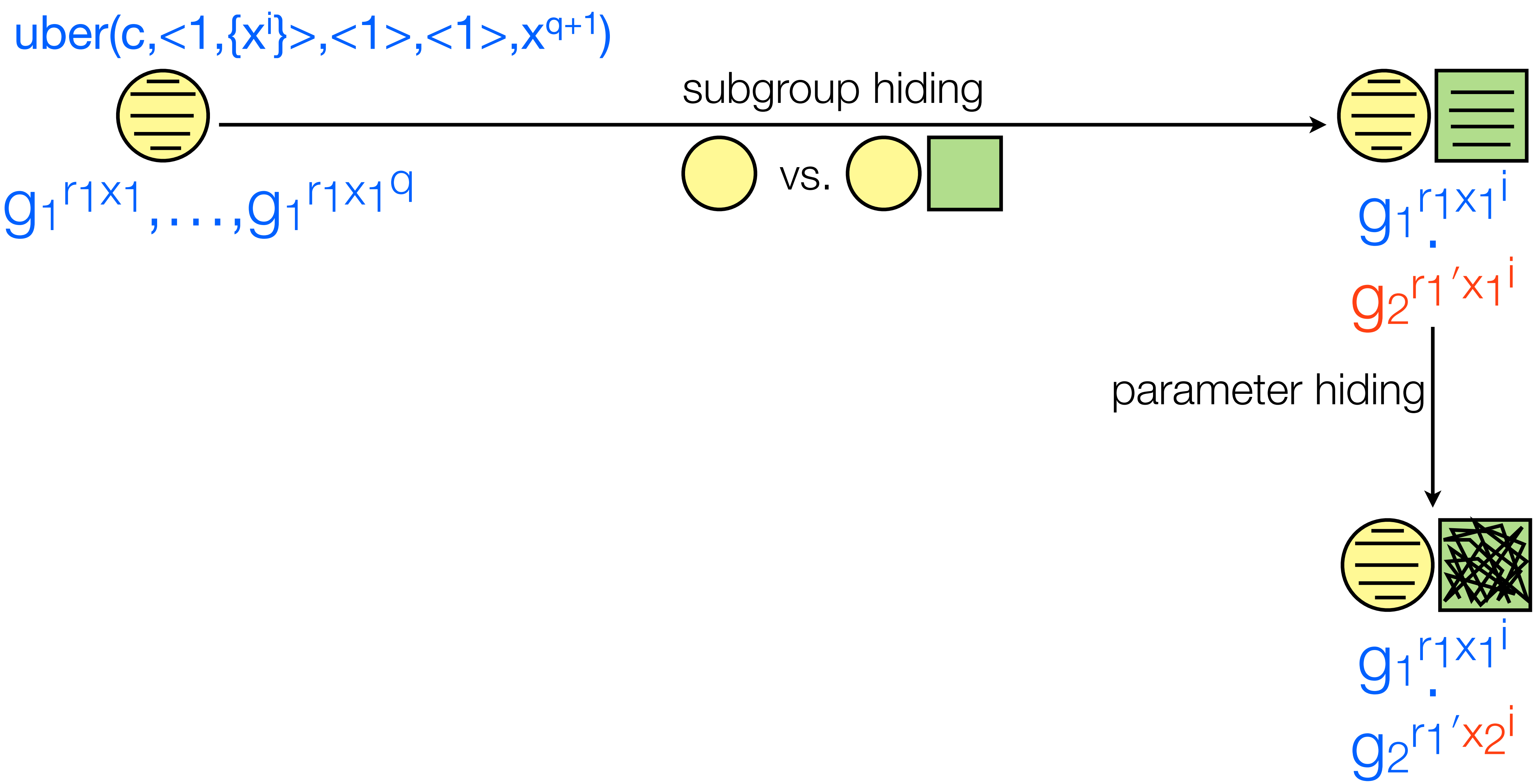

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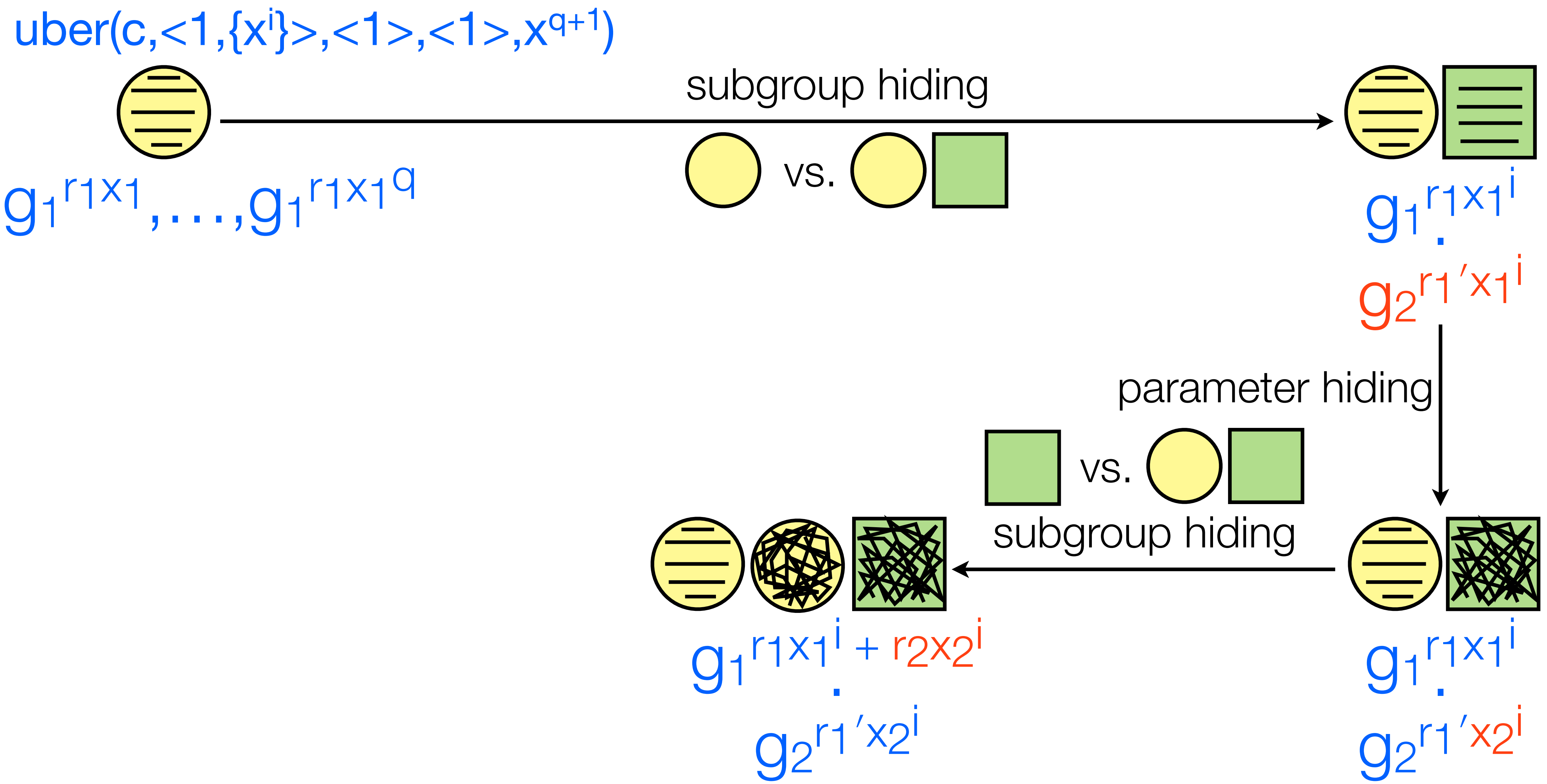

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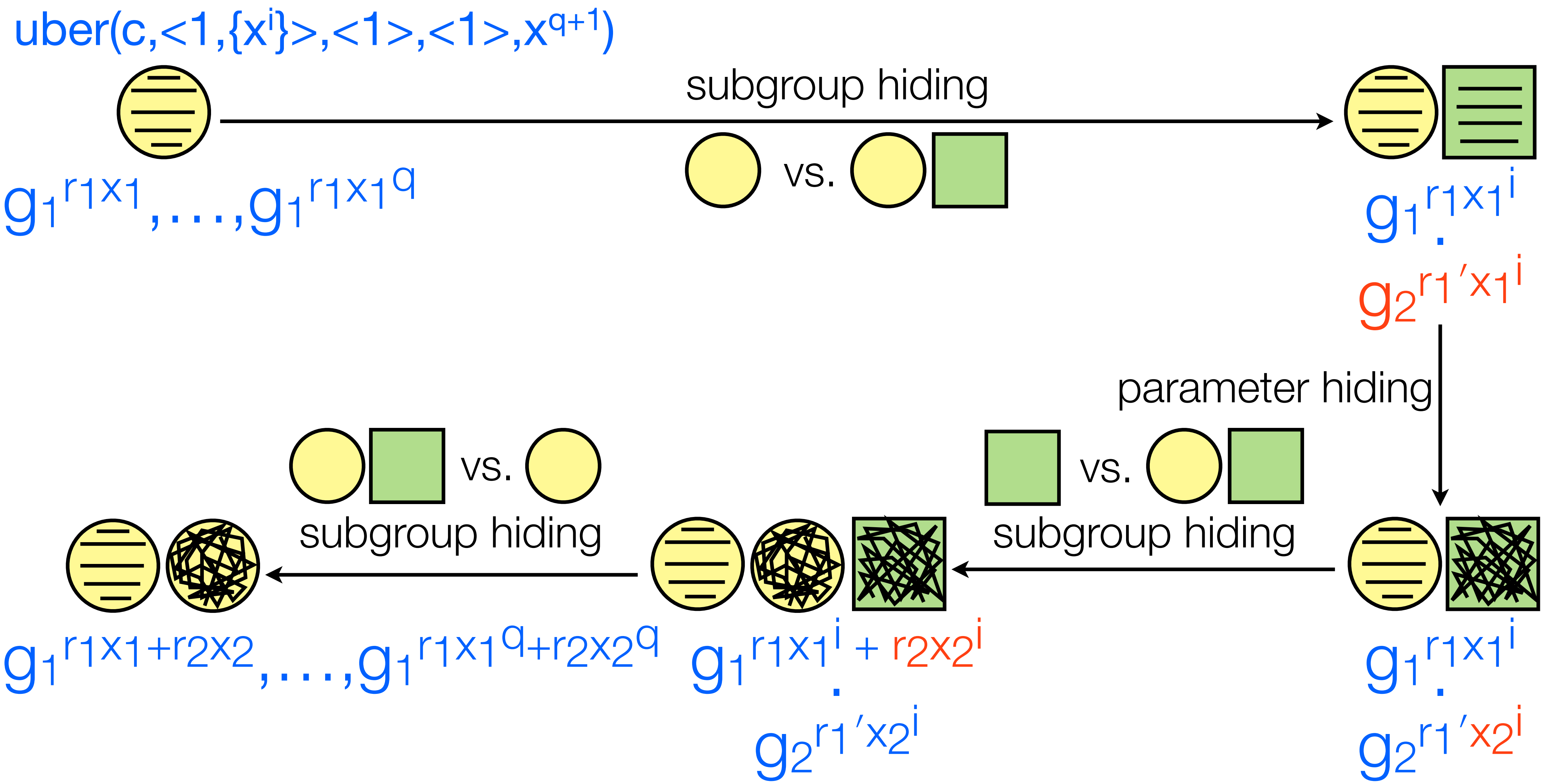

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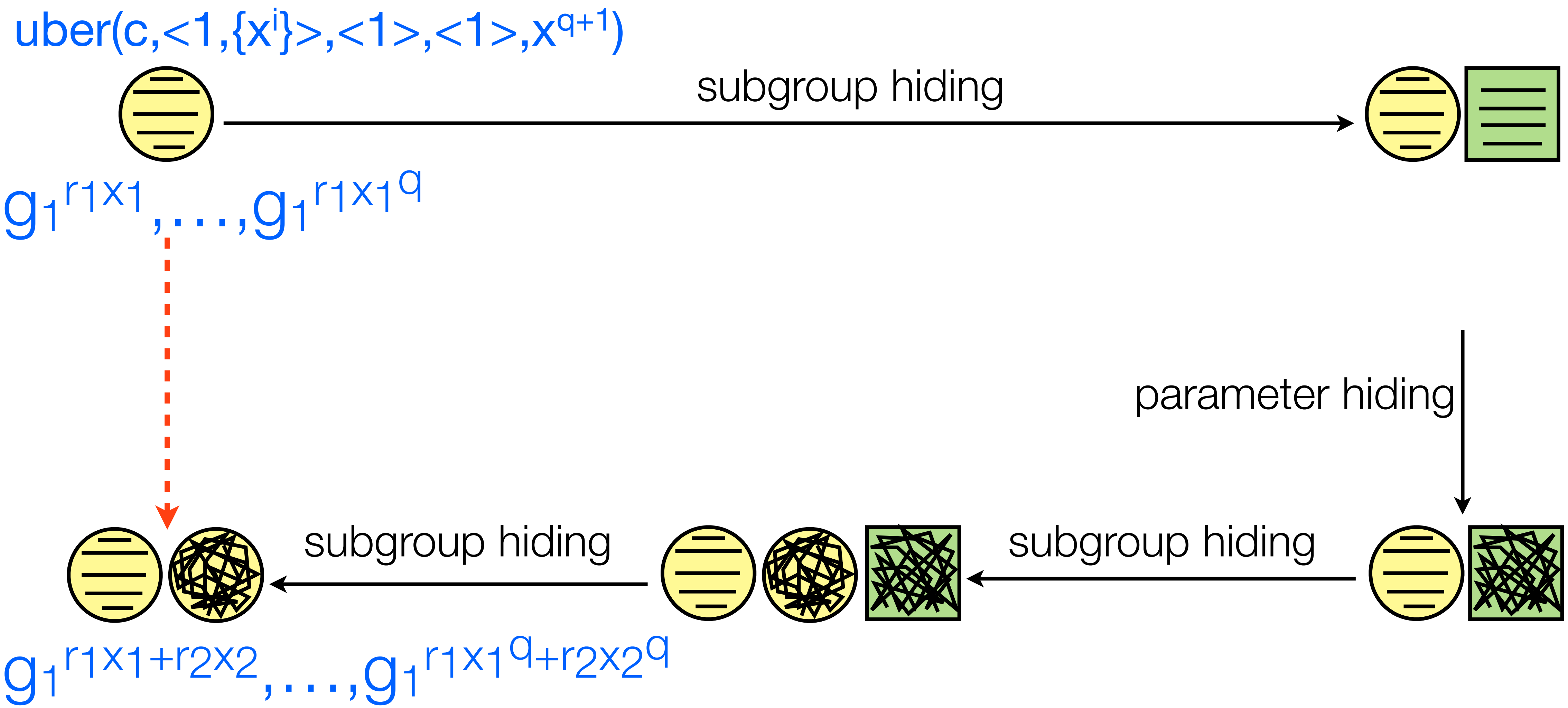

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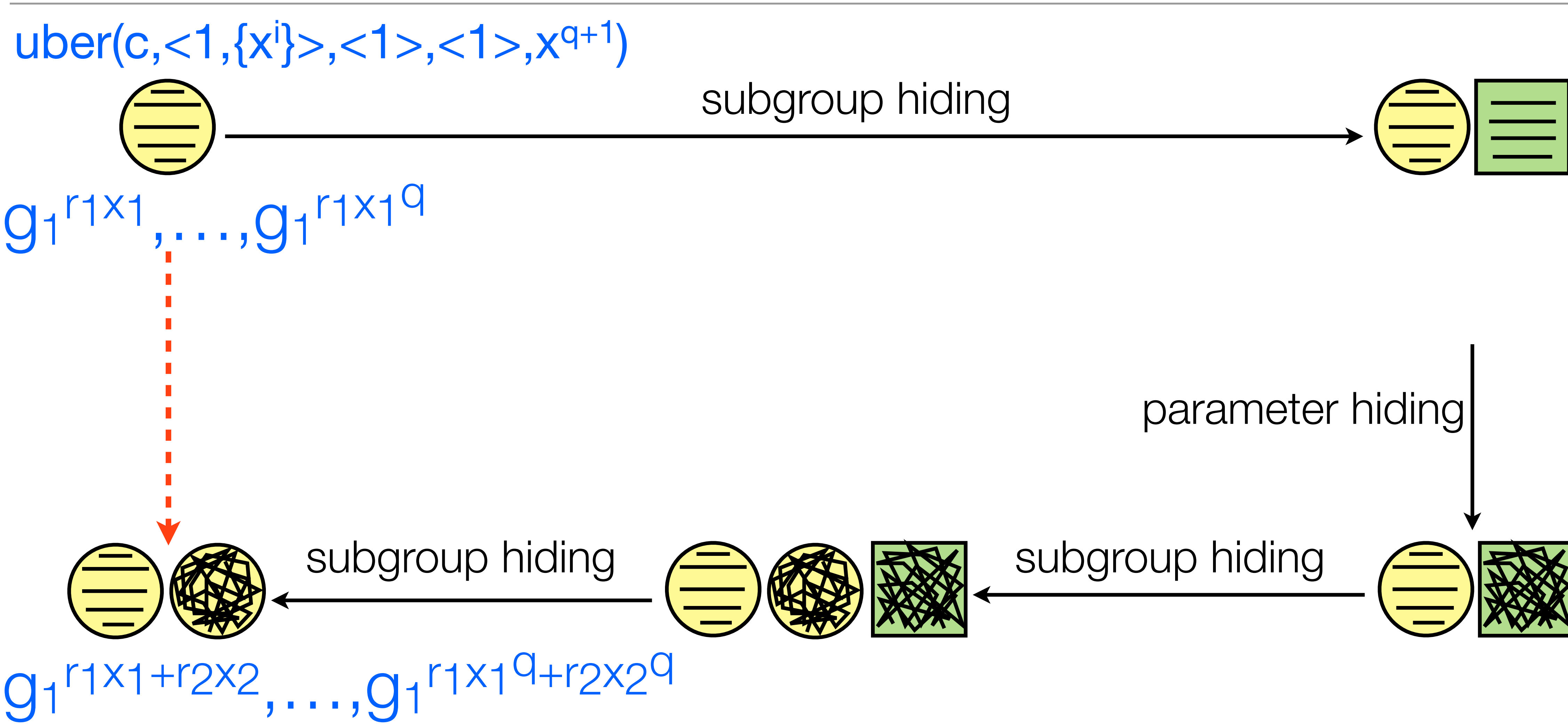

$g_1^{\sum r_kx_k},\ldots,g_1^{\sum r_kx_k^q}$

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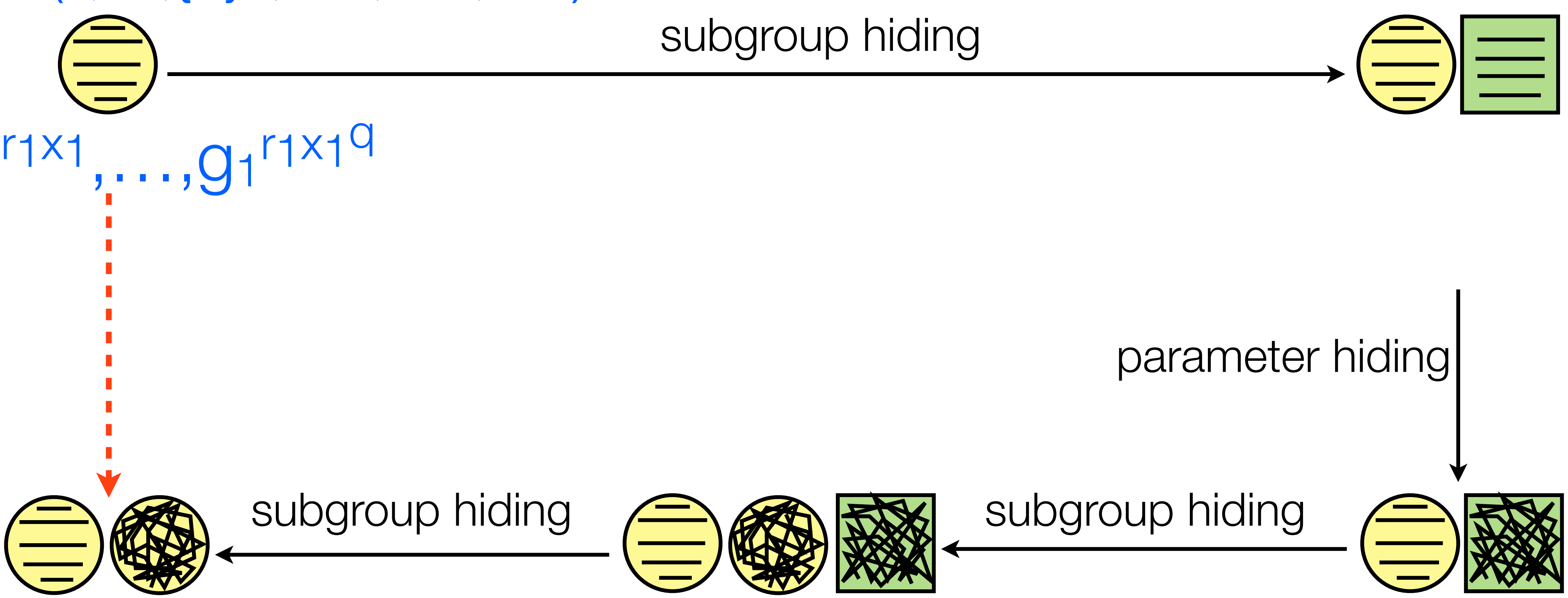

$g_1^{r_1x_1},\ldots,g_1^{r_1x_1^q}$

$g_1^{r_1x_1+r_2x_2},\ldots,g_1^{r_1x_1^q+r_2x_2^q}$

$g_1^{\Sigma r_kx_k},\ldots,g_1^{\Sigma r_kx_k^q}$

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Applying dual systems to exponent q-SDH

$\text{uber}(c,R,<1,\{x^i\}>,<1>,x^{q+1}) \rightarrow \text{uber}(\ell c,<1,\{\sum r_k x_k{}^i\}>,<1>,<1>,\sum r_k x_k{}^{q+1})$

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$$\begin{bmatrix} r_1 & r_2 & \dots & r_\ell \end{bmatrix} \begin{bmatrix} 1 & x & . & x^q & x^{q+1} \\ 1 & x_2 & . & x_2^q & x_2^{q+1} \\ . & & . & & . \\ . & & & . & . \\ 1 & x_\ell & . & x_\ell^q & x_\ell^{q+1} \end{bmatrix}$$

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So A is really given

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2. transition to SF version
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Applying dual systems to exponent q-SDH

$$\begin{bmatrix} r_1 & r_2 & \dots & r_\ell \end{bmatrix} \begin{bmatrix} 1 & x & . & x^q & x^{q+1} \\ 1 & x_2 & . & x_2{}^q & x_2{}^{q+1} \\ . & & . & & . \\ . & & & . & . \\ 1 & x_\ell & . & x_\ell{}^q & x_\ell{}^{q+1} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_\ell \end{bmatrix}$$

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Vandermonde matrix,
so if ℓ=q+2 this is invertible

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Applying dual systems to the uber-assumption

More generally, this is true if

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limitation $\longrightarrow$ 2. $S = T = \langle 1 \rangle$

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Broader classes of assumptions

Dodis-Yampolskiy PRF

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Strengthening our results

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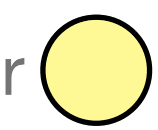
Remember that we needed two types of subgroup hiding …

…even when given a generator for 

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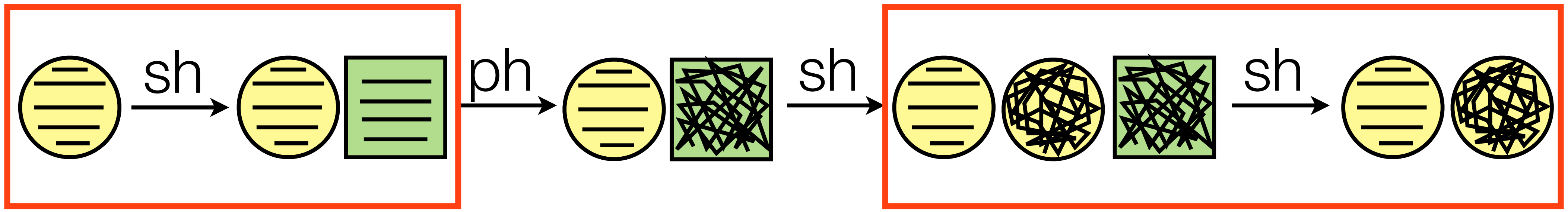

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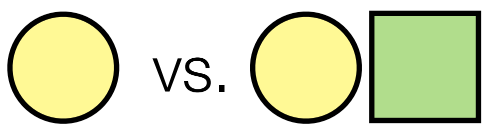
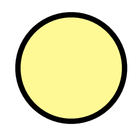


…even when given a generator for 

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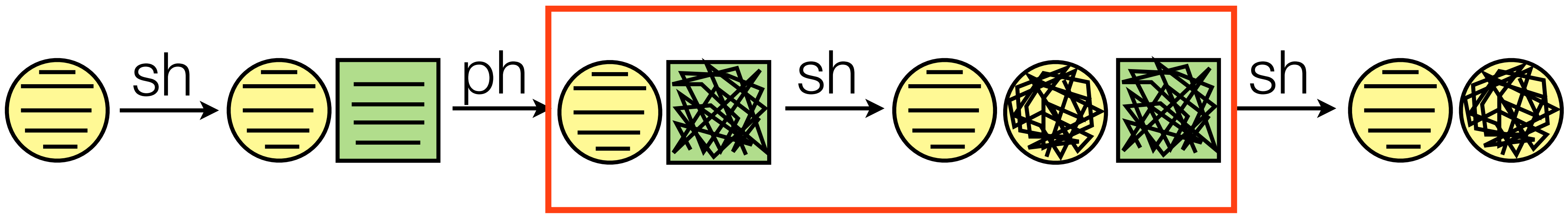

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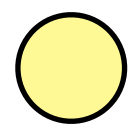
…even when given a generator for 

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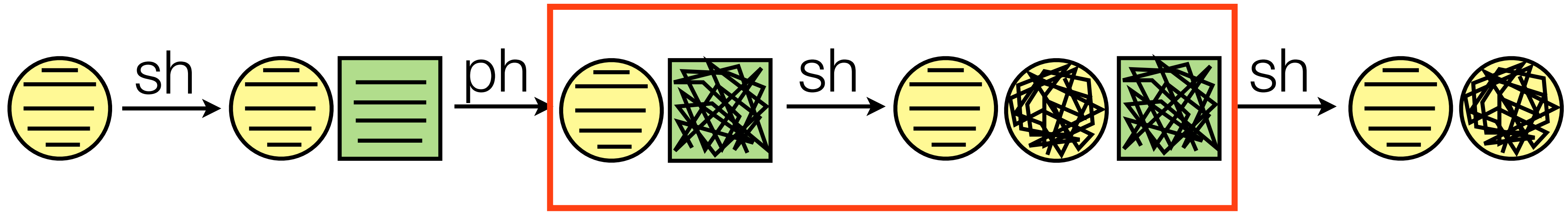

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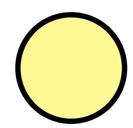
…even when given a generator for 

This restricts us to “one-sided” assumptions

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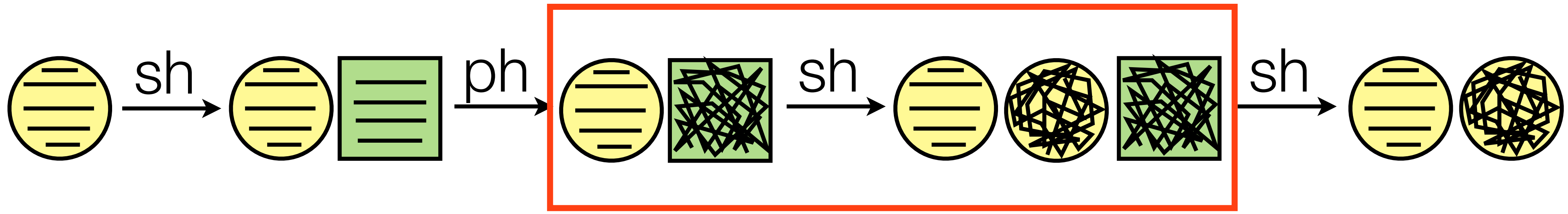

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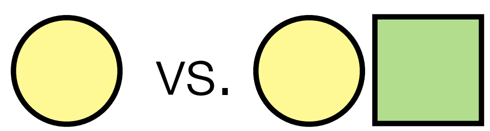
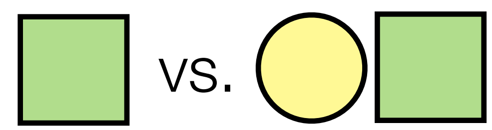
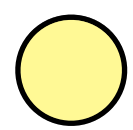




…even when given a generator for 

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[eq-SDH]

$$(g,g^x,\ldots,g^{x^q}) \to g^{x^{q+1}} \text{ or random}$$

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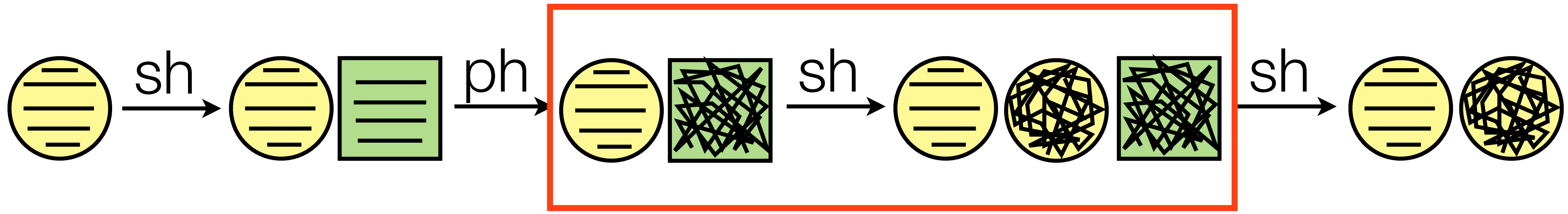

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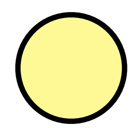
…even when given a generator for 

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[eq-SDH]  $(g, g^x, \ldots, g^{x^q}) \rightarrow g^{x^{q+1}}$ or random

[q-SDH]  $(g, g^x, \ldots, g^{x^q}, h^x) \rightarrow$ compute $(c, g^{1/x+c})$

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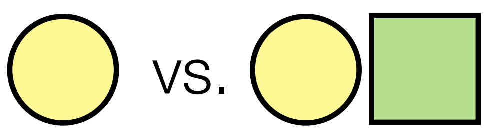

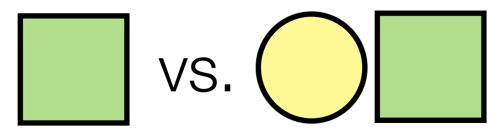

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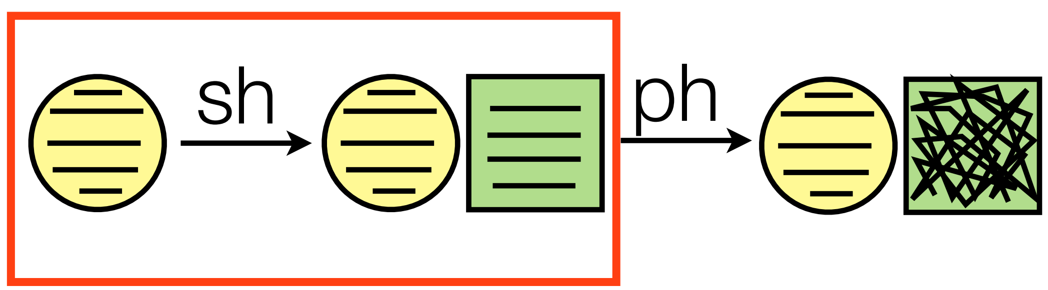

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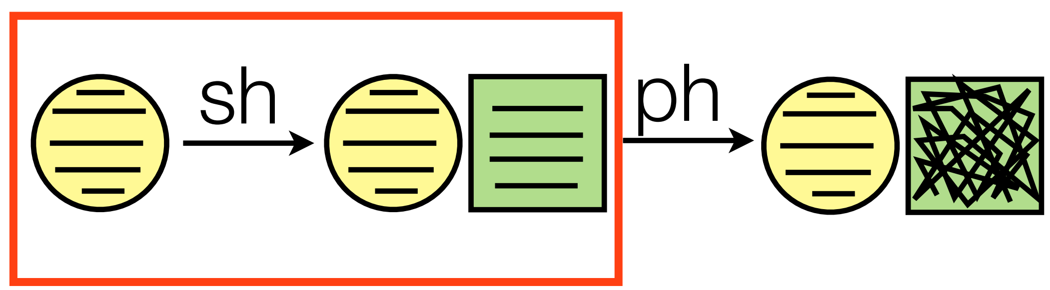

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This implies (for example) that q-SDH [BB04] follows from subgroup hiding....

…and so does everything based on q-SDH (like Boneh-Boyen signatures)*

*when instantiated in asymmetric composite-order groups [BRS11]

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Full version!: `cs.ucsd.edu/~smeiklejohn/files/eurocrypt14a.pdf`

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Thanks! Any questions?